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Probing Assembly Models: Exploring Innovations in Models with Perfect Complementarity

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Abstract

This paper introduces and scrutinizes three groundbreaking models aimed at probing the intricacies of duopoly dynamics within perfect complement factors: (1) the spontaneous assembly model, (2) the non-spontaneous assembly model, and (3) the sequential spontaneous assembly model. This study delves into oligopoly with perfect complementarity and makes the following discoveries. First, the renowned and significant quantity competition model, the Cournot model, is equivalent to the price competition models studied in this research, the spontaneous assembly model and the non-spontaneous assembly model using ECPR. Second, the renowned and significant quantity competition model, known as the chain monopoly model, aligns with the price competition model explored in this study, termed the sequential spontaneous assembly model. Third, the three price competition models examined in this study are found to be as follows. Initially, a comprehensive investigation of the spontaneous assembly model lays the basis for a compelling argument, demonstrating that the equilibrium price of the 'system goods' in duopoly surpasses that of the monopoly scenario. Subsequently, we meticulously establish the mathematical equivalence between the non-spontaneous assembly model, using the 'efficient component pricing rule', and the spontaneous assembly model. Our focus then shifts to the sequential spontaneous assembly model, where a meticulous comparison with the simultaneous model unveils elevated equilibrium prices for the 'system goods'. In particular, we elucidate that those industrial profits are based on the price elasticity of demand, with this sequential model yielding higher profits under conditions of inelastic demand.

Keywords: *Perfect Complement Factors, Spontaneous Assembly Model, Non-Spontaneous Assembly Model, Sequential Spontaneous Assembly Model, Efficient Component Pricing Rule (ECPR), Chain Monopoly.*
JEL Classification: L12, L13, L22.

Introduction

The origin of the "quantity competition" model under duopoly traces back to Cournot's seminal work in 1838, titled "Researches into the Mathematical Principles of the Theory of Wealth". Despite this historical significance of Cournot's contribution, a considerable portion of the content within the publication has been overlooked. This oversight prompts a reexamination of the intriguing model known as the 'copper-zinc duopoly', where one company monopolizes copper production, and another company monopolizes zinc production. In particular, copper or zinc alone has minimal value for consumers; however, its amalgamation results in brass, a valuable commodity. In contemporary terms, we conceptualize brass as a 'system goods', with copper and zinc serving as its perfect complement factors. The production of 1 unit of brass involves perfectly competitive downstream firms purchasing 1 unit of zinc and 1 unit of copper from upstream monopolistic zinc and copper firms, respectively, to manufacture the final product. Monopolistic zinc and copper companies engage in quantity competition, known as the famous copper-zinc duopoly.

Another well-known model of "quantity competition" is the 'chain monopoly', which involves two entities: the upstream monopoly and the downstream monopoly. In the chain-monopoly model, the upstream monopolist is engaged in the production of an intermediate product. Subsequently, the downstream monopolist acquires the intermediate product from the upstream monopolist to further manufacture the final product. This pattern of upstream and downstream monopolies engaging in quantity competition is called the 'chain monopoly'.

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The rationale behind this study can be encapsulated as follows: (1) Previous research has left gaps in our understanding regarding the interaction between quantity competition and price competition in oligopolistic markets. This has led to an interest in investigating whether other forms of price competition could be analogous to two well-known forms of quantity competition: Cournot's quantity competition and chain monopoly quantity competition. (2) There is a dearth of literature on perfect complement oligopolistic markets, leaving the nature of equilibrium prices undetermined. Consequently, there is an interest in deeper delving into the properties of equilibrium prices under price competition in perfect complement scenarios. (3) Can the characteristics outlined in (1) and (2) be combined to establish a model equivalence relationship?

Therefore, spurred by the above research motivations, this study aims to investigate further the following research objectives: (1) To dive into the exploration of oligopolistic markets under perfect complement conditions, a topic that has received scant attention thus far, and to understand the characteristics of equilibrium prices in both static and dynamic models. (2) To examine whether specific price competition models are analogous to the equilibrium outcomes of two well-known and classical quantity competitions, namely Cournot's and chain monopoly quantity competitions.

The inherent synergies observed in materials such as brass, copper, and zinc find common parallels in the contemporary landscape. This phenomenon extends to modern technologies, where a computer comprises both hardware and software components. The intricate interplay of complementary factors is a characteristic feature of various 'system goods'. Despite the recognition of this industrial structure by economists, who have employed intricate models to analyze aspects like standardization, firm compatibility choices, and network externalities, there remains a surprising dearth of research directly examining the duopoly of complementing factors.

In the following a comprehensive examination of the literature related to the extension and analysis of Cournot's seminal work (1838) on perfect complementarity in production is presented.

Chen (2005) employed Cournot's (1838) framework to establish three papers through the perfectly complementary characteristics of the elements. The thesis includes three papers. The first paper is "The Oligopoly of Perfect Complementary Components." The second paper is "Perfect Complementary and Mixed Oligopoly of Public and Private Firms." The third paper, "System Intermediate Goods and Optimal Final Goods Export Trade Policy," is similar to our research. But this thesis is written in traditional Chinese.

Sonnenschein (1968) established a formal correspondence between Cournot's duopoly and complement monopoly theories, underscoring their shared traits despite divergent symbolic interpretations. His paper aims to clarify this equivalence, emphasizing similarities, and critically analyzing duopoly theory. Cournot's duopoly theory explores competition dynamics among producers offering identical products. On the other hand, his complement monopoly theory explores situations in which products from two producers generate utility solely when amalgamated in a predetermined proportion to create a unified commodity. Edgeworth's observation further highlights this contrast, demonstrating the absence of dual pricing in duopoly situations and the independent nature of quantities in complementary monopoly contexts (Edgeworth, 1925, p. 122).

Amir and Gama (2019) conducted a thorough examination of Cournot's complementary monopoly model, delving into market entry effects and model uniqueness. Their work offers a nuanced characterization, challenging the assumption that it is the dual counterpart to the traditional Cournot oligopoly. The findings reveal that non-zero production costs disrupt duality, implying that an oligopoly with perfect complements may exhibit global strategic complementarity, even with production costs. Proposition 1 of our research establishes that the total price of the duopolistic is greater than that of the monopolistic price. This result can also be found in Proposition 9 of Amir and Gama (2019) for $n > 2$ perfect complements. This result is the same as the finding of Chen (2005), so the articles of these three articles are consistent.

Spence (1976) found that prices often fail to capture enough social benefits for valuable products, revealing key insights: the market system may not consistently generate the optimal product range; products tied to specialized interests may remain underdeveloped due to sellers' incapacity to capture benefits; sellers may

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resort to discrimination to appropriate more benefits, potentially expanding choices but incurring efficiency costs. Multi-product firms tend to resemble monopolistic competition in their selection patterns, while monopolies may restrict substitutes. In summary, this analysis highlights various phenomena and urges scrutiny of potential market failures.

Bertoletti (2022) identified that the Cournot oligopoly model, characterized by perfect complements in a quantity setting, attains a distinctive Nash equilibrium with zero quantities under mild conditions. This observation suggests inherent limitations on the availability of perfectly complement goods, unless the market operates under conditions of perfect competition or monopoly.

Goltsman and Pavlov (2012) advanced literature by studying communication in a Cournot duopoly with unverifiable costs. Cheap direct talk lacks substance, but using a third party enables informative communication without commitments. Their proposed mechanism ensures informationally and achieving Pareto dominance.

Grisáková and Štetka (2022) explored dynamics in a tri-firm oligopolistic market using the Cournot model, introducing realism by differentiating firms based on production. The article presented a specialized oligopolistic model accommodating three types of expectations, examining stability in a three-company oligopoly with partial differentiation over an infinite time horizon.

Quint (2014) extensively studied price competition dynamics in markets where final products are assembled from factors provided by different monopolists. Research established criteria for a discrete-choice demand system, revealing log-concave price-demand relationships and increasing price differences among products. This framework extends insights from basic models, including mergers or changes in marginal costs on prices, to this nuanced context.

Vives (2018) extensively explored the lattice-theoretic approach in oligopoly game analysis, providing valuable insights beyond strategic complementarities. This versatile method enables comparative statics even in scenarios with coexisting patterns of complementarity and substitutability.

Naimzada and Pireddu (2024) associated market volatility with endogenous fluctuations from non-linearities. They replaced the Mamada and Perrings (2020) linear rule with a sigmoid mechanism, studying quadratic emission charges in a Cournot duopoly. The sigmoid nonlinearity ensured stability in output variations. Extending to differentiated products, two dynamics investigations evaluated environmental policies in unstable Nash equilibrium. Comparative studies showed the effectiveness of policies in various scenarios, offering insights into adjusting the sigmoid mechanism for stability and pollution control.

Chen, Chow, and Liu (2023) used imitation and replicator dynamics to investigate the progression of firms that employ Cournot and Bertrand strategies in duopoly scenarios. They found globally asymptotically stable limits, dominated by scenarios identified in replicator dynamics. Imitation dynamics primarily led to the latter two equilibria. The stability of replicator dynamics, affected by product distinctiveness and the characteristics of items in linear demand and cost settings, indicated that evolutionarily stable tactics in dual market competitions with varied merchandise might not necessarily be the stable confines of replicator dynamics but could attain stability with uniform products. Using the models in enterprises engaged in the production of goods across a spectrum of quality levels elucidated fascinating deviations when juxtaposed with static models.

Dias Júnior, Santos, Soubeyran, and Souza (2023) introduce novel iterative techniques for addressing quasi-equilibrium problems in Hilbert spaces. Demonstrating convergence, they justify a novel perturbation choice, especially in the Cournot duopoly model. A novel approach to QEP improves conceptual clarity and mathematical rigor, facilitating the derivation of the perturbation function. Computational simulations robustly corroborate the efficacy of the proposed methodologies.

Although existing research has significantly contributed to understanding the dynamics of complementary elements, the specific focus on the duopoly of perfect complement factors has been notably understudied.

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Recognizing this void in theoretical development, our study seeks to bridge this gap by proposing three models dedicated to the analysis of duopoly within the realm of perfect complement factors. Although the phenomenon of complement factors has attracted significant academic interest, as evidenced by the studies of Hecking and Panke (2014) and Matsushima and Mizuno (2012, 2013), direct investigation of duopoly dynamics within the framework of perfect complement factors presents a promising avenue for further research.

In this study, we explore the nuances of a foundational model referred to as the spontaneous assembly model (referred to as Model A). This model focusses on a 'system goods' consisting of two perfect complement factors, each produced exclusively by a monopolistic firm. In the consumer-orientated context, individuals have the choice to directly purchase these factors from the respective companies and then participate in spontaneous assembly to acquire the final 'system goods'.

Our investigation places significant emphasis on the equilibrium pricing dynamics within Model A, aligning with the foundational principles delineated in Cournot's model. Our findings underscore that the optimal pricing of the 'system goods', as shown in Model A (also reflected in Cournot's framework), exceeds the equilibrium pricing observed in a monopolistic environment. This observation has profound implications for understanding market behaviors and consumer preferences within the framework of spontaneous assembly models.

Model B, identified as the non-spontaneous assembly model, unfolds a scenario where assembly technology is exclusively held by firms, barring consumers from this capability. In this model, firms participate in intercomponent trading, subsequently assembling the final 'system goods' for consumer sales. It is interesting to note that, under the Effective Component Pricing Rule (ECPR), we establish the mathematical equivalence between Model B and Model A, the spontaneous assembly model.

The ECPR has emerged as a central locus within modern regulatory economic discourse, particularly in the domain of examination related to access pricing strategies. Pioneering contributions by Willig (1979) and Baumol (1983) have delineated the ECPR, highlighting access pricing as the aggregate of the directly incremental cost per unit of input and the foregone revenue sacrificed by the provider of the resource when allocating a unit of input. Building upon this foundational work, Larson and Lehman (1997) provided insight into the conditions under which the ECPR aligns with Ramsey pricing rules. Furthermore, Larson (1998) has demonstrated the efficacy of the ECPR in enhancing economic efficiency.

The examination of these models and the inherent implications of the ECPR make substantial contributions to our understanding of regulatory economics. This research offers valuable information on the characteristics of access pricing and its potential influence on economic efficiency.

In the realm of duopoly dynamics surrounding complementary factors, our attention is directed towards the sequential spontaneous assembly model, denoted as Model C. This model intricately explores the sequential nature of duopoly, wherein one factor producer takes on the role of the leader, setting the initial price for its factor. The subsequent participant, acting as a follower, then formulates the pricing strategy for its own factor. Consequently, consumers make informed choices regarding the acquisition of individual factors and the assembly of the final 'system goods.'

A crucial facet of our investigation lies in recognizing and validating the benefits of being an early adopter within the context of Model C. By contrasting it with the concurrent model (Model A), we reveal a significant discrepancy in the price equilibrium of the 'system goods'. Particular, the sequential model (Model C) manifests an elevated equilibrium price for 'system goods'. Moreover, our analysis delves into the intricacies of industrial profits, demonstrating that the sequential model yields higher profits under conditions of inelastic demand and lower profits when demand exhibits elasticity.

This investigation makes a substantial contribution to the comprehensive understanding of duopoly dynamics, specifically by illuminating the consequences of sequential decision making in the context of perfect complement factors.

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In this study, we conclude our investigation by establishing the fundamental equivalence between the chain-monopoly model and Model C, this dynamic duopoly model of perfect complement factors. This finding is of considerable importance, as it not only reconciles the conceptual disparity between these two frameworks, but also presents a fresh lens through which to grasp the dynamics of interaction between monopolies in the supply chain.

The extensive corpus of literature exploring the complexities of chain monopoly market structures provides valuable insight. Building on this knowledge base, our analysis seamlessly extends into the realm of sequential duopoly, incorporating established principles derived from the study of double marginalization, vertical restrictions, and vertical integration. Through this approach, we enhance the depth of our investigation, tapping into the wealth of knowledge accumulated within the framework of chain monopolies.

This study makes a significant contribution to the ongoing discourse on perfect complement factors. It extends our understanding of market structures by providing a nuanced examination of the interplay between upstream and downstream monopolies.

This study unfolds in a meticulously structured manner, aiming to comprehensively explore the dynamics of perfect complement factors within the context of a duopoly framework. The following sections delineate the sequential progression in our analysis.

In Section 2, the foundational spontaneous assembly model is presented, establishing the foundation for our examination of equilibrium dynamics within a duopolistic framework. Section 3 extends our investigation to delve into the intricacies of the non-spontaneous assembly model, providing insights into the implications of this variation. Section 4 meticulously unravels the complexities of the sequential spontaneous assembly model, providing a comprehensive understanding of the dynamics inherent in a duopoly where one firm assumes the role of a leader, subsequently influencing the pricing strategies adopted by a follower. Section 5 presents insightful comparisons between the sequential spontaneous assembly model and the chain-monopoly model, shedding light on the nuanced interplay of market dynamics.

The concluding section consolidates our findings, offering a coherent summary of the insights and contributions presented throughout the paper.

A Comprehensive Analysis of a Spontaneous Assembly Model

The spontaneous assembly model, denoted Model A, addresses the conceptualization of a 'system goods', denoted as y, which consists of two complementary factors, x_1 and x_2. Each factor, x_i, is manufactured by a monopolistic entity i without incurring any manufacturing expenses. Consumers have the ability to assemble x_1 and x_2 into y utilizing the assembly function $y = \min\{x_1,\ x_2\}$. For simplicity, we assume that assembly expenditure is negligible (refer to Figure 1).

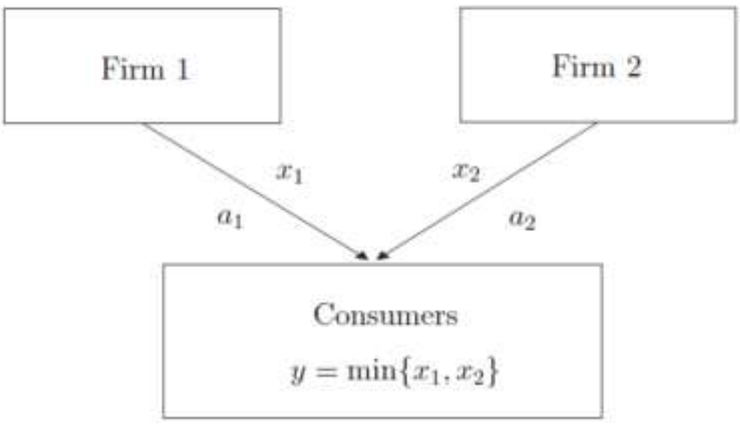

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Figure 1. Spontaneous Assembly Model

Let a_i denote the factor x_i price. Given that a consumer is required to remit $p_y = a_1 + a_2$ for the acquisition y of one unit, so the demand function for y be articulated as $D(p_y)$. We posit that this demand curve adheres to the principle of demand, specifically, $D' = dD(p_y)/dp_y < 0$. Subsequently, the profit function for firm i can be outlined in a subsequent manner.

$$\pi^i(a_1, a_2) = a_i D(p_y) \qquad (1)$$

The market functions based on the subsequent decision-making sequence. First, monopolistic firms determine their factor prices autonomously and concurrently. Subsequently, consumers make informed purchase decisions.

It is crucial to underscore that within the Cournot copper-zinc model of (refer to Figure 2), final consumers are not directly involved in the assembly process of the 'system goods'. Instead, they acquire these goods from the competitive retail market, where retailers source the necessary factors from monopolistic firms (upstream) to produce the final 'system goods'. The downstream perfectly competitive firms compete in quantity, with a production function given by $y = \min\{x_1, x_2\}$. Since the downstream firms operate under perfect competition, the price of brass is equal to the cost of purchasing intermediate goods; in other words, $p_y = a_1 + a_2$. Let (a_1^*, a_2^*) represent the optimal factor prices in Model A. The correspondence of the results suggests that the equilibrium price of the 'system goods' in Cournot's framework will be equivalent to $a_1^* + a_2^*$.

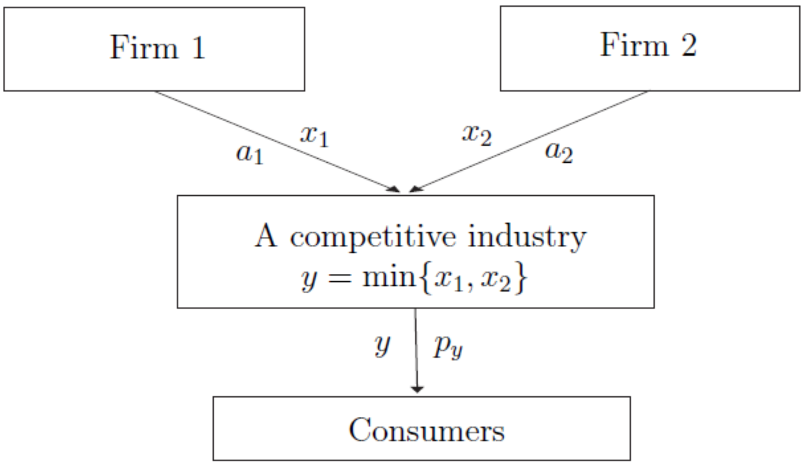

Figure 2. Cournot's copper-zinc Model

We seek to perform a comparative examination between Model A and the monopolistic setting. Clearly, in instances where both factors originate from a monopolistic source, the entity will select $p_y = p_{yM}$ to optimize the profit $\pi^m = p_y D(p_y)$. The ensuing result emerges from this comparative examination.

<u>Proposition 1</u> This 'system goods' equilibrium price $a_1^* + a_2^*$ of the spontaneous assembly model surpasses the optimal monopoly price p_{yM}.

Proof:

- Consider p_{yM} as the optimal price of monopoly, defined as $\arg\max_{p_y} p_y D(p_y) = p_{yM}$. We will demonstrate what $a_1^* + a_2^* \geq p_{yM}$. Assume the contrary, i.e., $a_1^* + a_2^* < p_{yM}$. Notably, due to symmetry, $a_1^* = a_2^*$. Consequently, we have $a_1^* + p_{yM}/2 = p_{yM}/2 + a_2^* < p_{yM}$. This implies

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$p_{yM}D(p_{yM}) < (p_{yM}/2)D(a_1^* + p_{yM}/2) + (p_{yM}/2)D(p_{yM}/2 + a_2^*)$. Moreover, following the characteristic of maximization, we obtain $(p_{yM}/2)D(a_1^* + p_{yM}/2) + (p_{yM}/2)D(p_{yM}/2 + a_2^*) \leq \max_{a_2} \pi^2(a_1^*, a_2) + \max_{a_1} \pi^1(a_1, a_2^*) = \pi^2(a_1^*, a_2^*) + \pi^1(a_1^*, a_2^*) = (a_1^* + a_2^*)D(a_1^* + a_2^*)$. Collecting these two outcomes, we deduce that $p_{yM}D\left(p_{yM}\right) < \pi^2(a_1^*, a_2^*) + \pi^1(a_1^*, a_2^*) = (a_1^* + a_2^*)D(a_1^* + a_2^*)$. Since this contradicts the optimal choice of the monopolist, it must hold that $a_1^* + a_2^* \geq p_{yM}$.

- Assume $a_1^* + a_2^* = p_{yM}$. The aggregation of the derivative conditions for $\max_{a_1} \pi^1(a_1, a_2^*)$ and $\max_{a_2} \pi^2(a_1^*, a_2)$ results in $D(a_1^* + a_2^*) + a_1^* D'(a_1^* + a_2^*) = 0$ and $D(a_1^* + a_2^*) + a_2^* D'(a_1^* + a_2^*) = 0$. Combining these equations yields $2D(p_{yM}) + p_{yM} D'(p_{yM}) = 0$. The monopoly price is $p_{yM} = \arg\max_{p_y} p_y D(p_y)$, and its first-order condition is $D(p_{yM}) + p_{yM}D'(p_{yM}) = 0$. Consequently, $2D(p_{yM}) + p_{yM}D'\left(p_{yM}\right) = D\left(p_{yM}\right) + p_{yM} D'(p_{yM}) = 0$. Given that this observation is in opposition to the condition $D(p_{yM}) > 0$, we thereby establish the conclusion of the demonstration. Therefore, the demonstration is complete.

The identification of Proposition 1 aligns remarkably with Bellaflamme and Peitz (2015: section 3.3.2), underscoring the significance and validity of this proposition. At initial inspection, the aforementioned findings may appear counterintuitive, as they suggest that "the more competition, the higher the prices". This outcome is not typical in markets where products are substitutes. However, in an oligopoly involving complementary products, a reduction in the price of one firm can stimulate increased demand for the other firm and neither firm is inclined to supply the public good. The positive externality, which can be internalized in a monopoly setting, leads to the observation of a lower price in a monopoly. It is essential to highlight that, in a monopoly, both consumer surplus and social welfare are elevated.

Examining Proposition 1 and its subsequent discussion underscores the crucial need for a thorough exploration of the economic implications arising from the seemingly counterintuitive result. In the spontaneous assembly model, where the equilibrium price surpasses the optimal monopoly price, a distinct dynamic is evident, shaped by product complementarity. Unlike traditional markets with substitutes, oligopolies involving complementary products may paradoxically experience elevated prices with increased competition. This phenomenon originates from the intricate interplay between firms and the positive externality linked to complementary goods.

When one firm lowers its price, it stimulates an increase in demand for the complementary product, initiating a mutually strengthening cycle. Importantly, neither firm is motivated to independently provide the public good, and this positive externality remains insufficiently internalized in a competitive setting. Consequently, the equilibrium price in the spontaneous assembly model surpasses the optimal monopoly price, challenging conventional expectations and prompting a deeper exploration of the underlying mechanisms and implications for consumer surplus, social welfare, and overall market efficiency in such economic contexts.

Based on the above analysis, under conditions of complete complementarity, the equilibrium outcomes of the spontaneous assembly model under price competition are equivalent to Cournot's model under quantity competition.

A Comprehensive Analysis of a Non-Spontaneous Assembly Model

In Model B, depicting a non-spontaneous assembly scenario in this context, it is postulated that the assembly capability lies exclusively with firms rather than consumers (refer to Figure 3). Firms engage in factor exchange amongst themselves and subsequently fabricate the 'system goods' for commercial dissemination to consumers. It is assumed that all firms operate under an identical manufacturing equation: $y = \min\{x_i, x_j\}$.

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The market functions on the basis of the following sequential procedure. Initially, firms engage in factor exchange among themselves through an efficient component pricing rule. Subsequently, in the secondary phase, firms disclose pricing information p_i, and consumers subsequently render their purchasing determinations.

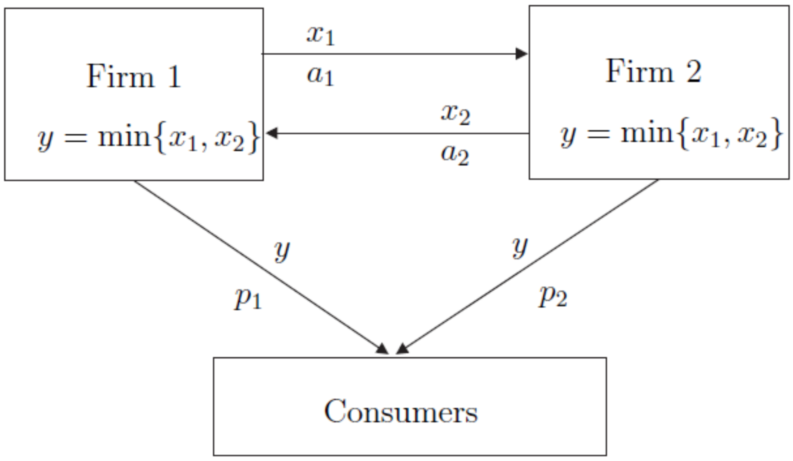

Figure 3. Non-Spontaneous Assembly Model

This framework is well suited for analyzing significant sectors such as global communication utilities, cellular telephony provisions, spoke-and-hub air transit frameworks, bilateral licensing of intellectual property rights, or any products characterized as 'system goods'. Using the global telecommunications sector as a case study, the 'system goods', an overseas telephone communication, encompasses the network services provided by telecommunication companies in various countries. It is evident that such network services function as ideal complements and consumers do not possess the capability to integrate them autonomously.

Using backward induction, our initial analysis should focus on the market for final products. It is important to emphasize the homogeneity of these final products.

In a market featuring undifferentiated products, competitive pricing drives the equilibrium market price in this model to $p_1 = p_2$. Given that firm i has the option to vend one unit of x_i to firm j at a price of a_i in the factor market, the associated cost for firm i to produce this factor becomes $p_i - a_j$. This is attributable to the fact that firm i loses the opportunity to sell one unit of 'system goods' to the consumer. Therefore, we derive $a_i = 0 + (p_i - a_j) = p_i - a_j$ through ECPR. The following proposition is presented.

Proposition 2 Using the non-spontaneous assembly model within ECPR yields the following equilibrium: $p_1 = p_2 = a_1 + a_2$. Therefore, the non-spontaneous assembly framework mathematically corresponds to the spontaneous assembly model.

Proposition 2 posits that in the equilibrium of the non-spontaneous assembly model using ECPR, the pricing variables converge as $p_1 = p_2 = a_1 + a_2$. This mathematical equivalence with the spontaneous assembly model is further substantiated by substituting this equilibrium result into the first-stage optimization problems of the firms, yielding the derivation of Equation (1). Therefore, from a mathematical perspective, the current model aligns with the spontaneous assembly paradigm. To enhance analytical simplicity and avoid unnecessary complexity, we assume that consumers have the ability to assemble the 'system goods' independently.

The mathematical equivalence explicated by Proposition 2 between the non-spontaneous assembly and spontaneous assembly models carries profound economic implications. It suggests that, despite disparate assembly mechanisms, the equilibrium outcomes and pricing dynamics in both models converge. This

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finding prompts an exploration of how consumer empowerment in the assembly of the 'system goods' shapes market behavior. The assumption of consumer assembly capacity introduces a novel dimension that could potentially influence consumer preferences, market competition, and overall welfare.

Based on the above analysis, under conditions of perfect complementarity, the equilibrium results of the non-spontaneous assembly model using ECPR under price competition are equivalent to Cournot's model under quantity competition. From the synthesis of Sections 2 and 3, it is deduced that the effect of Cournot's model under quantity competition is equivalent to both the spontaneous assembly model and the non-spontaneous assembly model using ECPR under price competition.

A Comprehensive Analysis of a Sequential Spontaneous Assembly Model

This sequential spontaneous assembly framework (Model C) is embodied in the ordered model. In recent history, the telecommunications sector has experienced deregulation, frequently coupled with liberalization or privatization, in various countries. This transition has resulted in a shift in the market organization of telecommunications, progressing from a monopolistic to a duopolistic state where enterprises offer interdependent network services. Nevertheless, it is customary for the existing company to take on the position of a pricing pioneer within the duopoly. Expanding this aforementioned model into a sequential game framework, one can comprehensively examine the dynamics of such a deregulated sector.

Let's consider a scenario in which firm 1 assumes the role of the price leader, initially setting its factor price a_1. Subsequently, firm 2, in the role of a follower, determines the factor price a_2. Once these factor price quotations are established, consumers decide on their purchases. It is assumed that consumers possess the ability to autonomously compile the 'system goods'. Consider $r_i(a_j)$ represent firm i's reaction function. Optimization problems for firms can be formulated in the subsequent manner.

$$\max_{a_1} \pi^1 = a_1 D(a_1 + r_2(a_1)) \qquad (2)$$

$$\max_{a_2} \pi^2 = a_2 D(a_1 + a_2), \text{ given } a_1 \qquad (3)$$

It is important to recognize what the existing valuation of the 'system goods' is determined by $p_y = a_1 + a_2$. Employing the retrospective inference approach, our analysis starts with the examination of the optimization problem of firm 2. Through straightforward algebraic calculations, we derive the first-order condition that the reaction function of firm 2, denoted as $a_2 = r_2(a_1)$, must satisfy.

$$\pi_2^2 = \frac{\partial \pi^2(a_1,a_2)}{\partial a_2} = D(a_1 + a_2) + a_2 D'(a_1 + a_2) = 0 \qquad (4)$$

The subsequent second-order condition is articulated in the following manner.

$$\pi_{22}^2 = \frac{\partial^2 \pi^2(a_1,a_2)}{\partial {a_2}^2} = 2D'(p_y) + r_2(a_1) D''(p_y) \leq 0 \qquad (5)$$

The gradient of the reaction function of firm 2 is also obtained through the following derivation.

$$r_2'(a_1) = -\frac{\pi_{21}^2}{\pi_{22}^2} = -\frac{\partial^2 \pi^2 / \partial a_1 \partial a_2}{\partial^2 \pi^2 / \partial {a_2}^2} = -\frac{D'(p_y) + a_2 D''(p_y)}{2D'(p_y) + a_2 D''(p_y)} \qquad (6)$$

To obtain more specific results, we introduce the additional assumption that $\pi_{ij}^i < 0$. Given that $\pi_{ii}^i = \pi_{ij}^i + D'(p_y)$ and $D'(p_y) < 0$, it directly implies that $|\pi_{ii}^i| > |\pi_{ij}^i|$, for all $i \neq j$. The initial assumption of $\pi_{ij}^i < 0$, coupled with the resulting observation $|\pi_{ii}^i| > |\pi_{ij}^i|$, collectively indicates that the gradient of the reaction function of firm 2 is a minus number and exceeds -1, that is, the absolute value is less than 1.

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The negative value of the gradient sign in the reaction function of firm 2 indicates that the pricing of the factors of the firms is strategic substitutes when $\pi_{21}^2 < 0$. This inference is particularly pertinent in the context of a duopoly comprising complementary products, where firms set prices strategically. The observation that the slope is less than 1 indicates that self-imposed price adjustments by a firm exert more influence than the impact of changes in the prices of the other firm.

This study will contrast the results of the sequential spontaneous assembly model with those of the non-sequential spontaneous assembly model (or non-spontaneous assembly model). Consider a_L^*, a_F^*, and a_A^* as the optimal factor prices determined by the leader, the follower in the sequential spontaneous assembly model, and each firm in the spontaneous assembly model, respectively. Similarly, denote p_{yC}^* as the optimal system price in the sequential spontaneous assembly model and p_{yA}^* as the optimal system price in the spontaneous assembly model. This comparative examination of these equilibriums is presented below.

Proposition 3 Comparing the equilibrium prices of two models, the sequential spontaneous assembly model (Model C) and the static spontaneous assembly model (Model A) or non-spontaneous assembly model (Model B), yields the following properties:

1. $a_L^* \geq a_A^* \geq a_F^*$.

2. $p_{yC}^* \geq p_{yA}^*$.

Proof:

Part 1:

- Step 1: Our objective is to demonstrate that $r_1(a_F^*) \leq a_L^*$. Let us consider the contrary scenario where $r_1(a_F^*) > a_L^*$. Given the negative gradient of the response function of firm 2, $r_2\big(r_1(a_F^*)\big) < r_2(a_L^*) = a_F^*$. Consequently, the following chain of inequalities emerges: $\pi^1(a_L^*, a_F^*) \leq_1 \pi^1(r_1(a_F^*), a_F^*) <_2 \pi^1(r_1(a_F^*), r_2(r_1(a_F^*)))$. Inequality (1) is justified by the characteristic of the reaction function, while inequality (2) is supported by the declining nature of the demand function $D(a_1 + a_2)$ relative to a_2. It's noteworthy that $(a_L^*, a_F^*) = (a_L^*, r_2(a_L^*))$ signifies the equilibrium state in Model C. Consequently, a paradox arises, compelling the deduction that $r_1(a_F^*) \leq a_L^*$.

- Step 2: Given the leader's prerogative to set the equilibrium value in Model A, identified as (a_1^*, a_2^*), its ensuing equilibrium gain in Model C exceeds that of any single firm in Model A. Formally, this is articulated as $\pi^1(a_L^*, a_F^*) \geq \pi^1(a_1^*, a_2^*)$.

- Step 3: Our objective is to establish $a_2^* \geq a_F^*$. Let us assume the contrary, i.e., $a_2^* < a_F^*$. Using the definition of the reaction function and the law of demand, it can be inferred that $\pi^1(a_1^*, a_2^*) = \pi^1(r_1(a_2^*), a_2^*) \geq \pi^1(r_1(a_F^*), a_2^*) > \pi^1(r_1(a_F^*), a_F^*) \geq \pi^1(a_L^*, a_F^*)$. This series of relationships contradicts the conclusion drawn in Step 2, necessitating the assertion that $a_2^* \geq a_F^*$ remains valid.

- Step 4: Based on the conclusions derived from Steps 3 and the characteristics exhibited by the reaction function, it can be established that $a_1^* = r_1(a_2^*) \leq r_1(a_F^*)$. Furthermore, given the results obtained in Steps 1, wherein $r_1(a_F^*) \leq a_L^*$, it follows that $a_1^* \leq a_L^*$.

- Step 5: It should be noted that due to symmetry, the prices of the equilibrium factor of model A are identical, denoted as $a_1^* = a_2^* = a_A^*$. Based on the outcomes obtained in Steps 3 and 4, it can be inferred what $a_L^* \geq a_A^* \geq a_F^*$.

Part 2: According to Eq. (6), it is evident that the magnitude of the gradient of the reaction function of firm 2 is below 1. As a result, it can be concluded what $a + r_2(a)$ constitutes a strictly increasing function. Given the prior derivation of $a_L^* \geq a_A^*$, it logically follows that $p_{yC}^* = a_L^* + r_2(a_L^*) \geq a_A^* + r_2(a_A^*) = p_{yA}^*$. Therefore, the demonstration is complete.

As previously observed, a positive spillover effect arises in a duopoly that features interdependent products. This leading entity, aware of the follower's pricing strategy dependent on its own initial pricing, strategically chooses a higher price to incentivize the follower to contribute to the provision of a public good. In the optimal state, the pricing of the 'system goods' in Model C matches or exceeds that of Model A. This indicates that the presence of a leading firm in such a market not only disadvantages competitors, but also has adverse effects on consumers. Our demonstrated finding that p_{yA}^* surpasses the monopolistic price (refer to Proposition 1) logically implies that p_{yC}^* must also exceed the monopolistic price.

Denote by π^{L*} and π^{F*} the equilibrium profits of the leading and following firms, respectively. Also, let π^{A*} represent the equilibrium profit of a firm in Model A. It's crucial to observe that p_{yC}^* and p_{yA}^*, falling outside the spectrum delineated by competitive and monopolistic prices, do not necessarily result in a commensurate increase in the profit of industry within these models. This evaluation of equilibrium profits is explained below.

Proposition 4 Comparing the equilibrium profits of two models, the sequential spontaneous assembly model (Model C) and the static spontaneous assembly model (Model A) or non-spontaneous assembly model (Model B), yields the following properties:

1. $\pi^{L*} \geq \pi^{A*} \geq \pi^{F*}$
2. $\pi^{L*} + \pi^{F*} \geq 2\pi^{A*}$ under the condition of inelastic demand.
3. $\pi^{L*} + \pi^{F*} \leq 2\pi^{A*}$ under the condition of elastic demand.
4. $\pi^{L*} + \pi^{F*} = 2\pi^{A*}$ under the condition of unitary elasticity in demand.

Proof:

Part 1:

- Step 1: Our objective is to demonstrate that $\pi^{L*} \geq \pi^{F*}$. It is crucial to highlight that $r_1(a_F^*) \leq a_L^*$ (specifically, an outcome derived from the findings of Step 1 as delineated in the proof provided in Proposition 3). Consequently, we can deduce that $\max_{a_2} \pi^2(r_1(a_2), a_2) \geq \pi^2(r_1(a_F^*), a_F^*) \geq \pi^2(a_L^*, a_F^*)$. By symmetry, $\max_{a_2} \pi^2(r_1(a_2), a_2) = \pi^{L*}$, and therefore $\pi^{L*} \geq \pi^{F*}$.

- Step 2: Observe what $a_1^* \leq a_L^*$ (specifically, an outcome derived from the results of Step 4 as demonstrated in Proposition 3). Taking advantage of the definition of the response function and the law of demand, what can establish that $\pi^2(a_1^*, a_2^*) = \pi^2(a_1^*, r_2(a_1^*)) \geq \pi^2(a_1^*, r_2(a_L^*)) = \pi^2(a_1^*, a_F^*) \geq \pi^2(a_L^*, a_F^*)$. Consequently, we have $\pi^{A*} \geq \pi^{F*}$.

- Step 3: Based on the outcomes of Steps 2 in the proof outlined in Proposition 3, it can be established that $\pi^{L*} \geq \pi^{A*}$. Furthermore, considering the results obtained in Step 1 ($\pi^{L*} \geq \pi^{F*}$) and Step 2 ($\pi^{A*} \geq \pi^{F*}$), we deduce that $\pi^{L*} \geq \pi^{A*} \geq \pi^{F*}$.

Part 2: Observe that $p_{yC}^* \geq p_{yA}^*$ (that is, the second item of Proposition 3). By differentiating the total profit $p_y D(p_y)$ with respect to p_y, we obtain $dp_y D(p_y)/dp_y = (1-\beta)D(p_y)$, where β denotes the demand function's elasticity. It becomes apparent that $\pi^{L*} + \pi^{F*} \geq 2\pi^{A*}$ when the elasticity is less than

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1; $\pi^{L*} + \pi^{F*} \leq 2\pi^{A*}$ when the elasticity exceeds 1; and $\pi^{L*} + \pi^{F*} = 2\pi^{A*}$ when the elasticity is unity. Therefore, the demonstration is complete.

The results outlined in the opening segment of Proposition 4 are consistent with those observed in most competitive duopoly models, diverging from the results of alternative complementary models. In contrast to entities in the concurrent model, the leader experiences enhanced outcomes, whereas the follower experiences a less advantageous situation. This suggests the existence of a temporal advantage within the sequential framework. The subsequent segments of Proposition 4 reveal what the comparative levels of profits of industry in the 2 frameworks are contingent on the elasticity. Based on the insights provided by Proposition 3, it can be deduced that the relationship between industry profit and the escalation (or reduction) of the price of 'system goods' may not consistently adhere to a monotonic pattern. However, a clearer conclusion can be drawn with an understanding of the elasticity of demand functions.

A Monopolistic Chain Model

In the context of the 'vertical monopoly chain', we examine two entities: the upstream monopolist (referred to as entity 1) and the downstream monopolist (referred to as entity 2). In this configuration, Entity 1 is involved in producing an intermediary commodity labeled as x_1. Following this, Entity 2 acquires the intermediary commodity x_2 to further produce the ultimate product y. It is apparent that the ultimate product comprises both the intermediary goods and the value added by Entity 2. Acknowledging that the value contributed by Entity 2 serves as a complement to the final product, we can argue that the vertical monopoly chain model bears a fundamental resemblance to the sequential assembly model.

A substantial volume of scholarly work has delved into the characteristic of a monopolistic chain market structure. Many academic investigations focus on themes such as double markup, vertical restrictions, and vertical consolidation. Nevertheless, to our current understanding, there has not been an acknowledgement what the monopolistic chain can be perceived as a sequential duopoly. These realizations offer a fresh perspective to understand the monopolistic chain model.

A current understanding regarding double markup, vertical constraints, and vertical consolidation should be able to be used smoothly to examine the sequential spontaneous assembly model. For instance, established findings from chain-monopoly research, demonstrating what the optimal price in the sequential spontaneous assembly model exceeds the monopolistic pricing, can be easily verified. Another notable example involves acknowledging that the leader in the sequential spontaneous assembly paradigm tends to utilize vertical constraint tactics, in line with our understanding of what the upstream entity in a monopolistic chain demonstrates, in a similar inclination.

The analysis carried out in Sections 4 and 5 demonstrates that the equilibrium outcomes of the traditional quantity competition chain monopoly paradigm align with those of our sequential spontaneous assembly framework in price competition.

Conclusions

This study aims to explore oligopolistic markets under the lesser-known condition of complete complementarity, focusing specifically on price competition. We strive to understand the uniqueness of equilibrium in such scenarios by analyzing static and dynamic models. Two classical quantity competitions are important: Cournot's quantity competition and chain monopoly's quantity competition. In particular, the equilibrium results of these two fundamental quantity competition models are equivalent to the price competition investigated in this study.

The most significant findings of this study make substantial contributions to academia. By delving into the concept of perfect complementarity, we discovered the following: First, the well-known Cournot model, which represents quantity competition, exhibits equilibrium equivalence to the spontaneous assembly model and the non-spontaneous assembly model using ECPR analyzed in this study under price

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competition. Second, the renowned chain monopoly model, representing quantity competition, demonstrates equilibrium equivalence to the sequential spontaneous assembly model examined in this study. These two significant discoveries bridge the gap between quantity and price competition within oligopolistic markets.

This study presents 3 frameworks to investigate the duopoly of perfectly complement factors. The primary framework, termed the spontaneous assembly model (Model A), investigates a scenario where a 'system goods' comprises 2 perfectly complementary factors, each manufactured by a monopoly firm. Consumers have the ability to integrate 2 factors into the 'system goods'. Additionally, our examination unveils that the optimal price of the 'system goods' in Model A surpasses the monopolistic price. This study demonstrates that the equilibrium of the spontaneous assembly model in price competition is equivalent to the Cournot quantity competition model in the context of the "copper-zinc duopoly."

The non-spontaneous assembly model (Model B) posits a scenario where only firms, rather than consumers, possess the assembly technology. Consequently, firms engage in factor trading among themselves, subsequently assembling the 'system goods' for consumer sale. Our analysis, supported by the extended efficient component pricing rule (ECPR), establishes the mathematical equivalence of this model with Model A. This study demonstrates that the equilibrium of the spontaneous assembly model and the non-spontaneous assembly model using ECPR in price competition is equivalent to the Cournot quantity competition model in the context of the "copper-zinc duopoly."

The sequential spontaneous assembly model (Model C) examines the dynamic competition within the complement duopoly. In this framework, one firm takes the lead as the initiator, setting the initial price for its factor, while the other firm, as the follower, determines the price of its factor. Consumers then decide to acquire factors and spontaneously assemble system goods. Our analysis verifies the existence of an initial advantage in this sequential spontaneous assembly model.

Furthermore, in contrast to the spontaneous assembly framework, it is evident that the optimal price of the 'system goods' in the sequential spontaneous assembly model is elevated. Additionally, this study infers that industry profit within the dynamic paradigm varies accordingly under conditions of inelastic (elastic) demand.

We unequivocally demonstrate the fundamental equivalence between the quantity competition chain monopoly model and the price competition sequential spontaneous assembly model. Drawing from the extensive literature on monopolistic chain structures, we use established insights regarding double marginalization, vertical constraints, and vertical consolidation to analyze the sequential duopoly paradigm that involves perfectly complementary factors. This revelation improves our understanding of the monopolistic chain and provides fresh insight into the interplay between upstream and downstream monopolies.

References

Amir, R. and Gama, A. (2019), "On Cournot's Theory of Oligopoly with Perfect Complements," mimeo.

Baumol, W. (1983), "Some subtle issues in railroad regulation," International Journal of Transport Economics, 10, 341-355.

Belleflamme, P. and Peitz, M. (2015), Industrial Organization. Markets and Strategies, second edition, Cambridge University Press, Cambridge (UK).

Bertoletti, P. (2022), "The dual of Bertrand with homogenous products is Cournot with perfect complements," Economic Theory Bulletin, 10,183-189.

Chen, C.H. (2005), Issues on Perfect Complementary Components. Unpublished doctoral dissertation, Graduate Institute of Industrial Economics, National Central University, Taiwan. The URL of the National Digital Library of Theses and Dissertations in Taiwan: https://hdl.handle.net/11296/g37s2p. Full text URL: https://drive.google.com/file/d/1awTkr4nJcaKoR77gVMro1hZl12xVOefU/view?usp=drive_link

Chen, H.C., Chow, Y., and Liu, S.M. (2023), "Dynamics of Cournot and Bertrand Firms: Exploring Imitation and Replicator Processes," Dynamic Games and Applications, https://doi.org/10.1007/s13235-023-00542-7.

Cournot A.A. (1971). Researches into the mathematical principles of the theory of wealth, 1838. with an essay Cournot and mathematical economics and a bibliography of mathematical economics. A.M. Kelley.

Dias Júnior, E.L., Santos, P.J.S., Soubeyran, A., and Souza, J.C.O. (2023), "On inexact versions of a quasi-equilibrium problem: a Cournot duopoly perspective," Journal of Global Optimization, DOI:10.1007/s10898-023-01341-5.

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https://ecohumanism.co.uk/joe/ecohumanism
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Edgeworth, F.Y. (1925), "The Pure Theory of Monopoly," Papers Relating to Political Economy. Vol. 1. London: Macmillan Co., for the Royal Econ. Soc.

Goltsman, M. and Pavlov, G. (2012), "Communication in Cournot oligopoly," Research Report, No. 2012-1, The University of Western Ontario, Department of Economics, London (Ontario).

Grisáková, N and Štetka, P. (2022), "Cournot's Oligopoly Equilibrium under Different Expectations and Differentiated Production," Games, 13(82): 1-17.

Hecking, H. and Panke, T. (2014). Quantity-setting Oligopolies in Complementary Input Markets - the Case of Iron Ore and Coking Coal. EWI Working Paper Series 14/06, Institute of Energy Economics at the University of Cologne (EWI).

Katz, M.L. and Shapiro, C. (1994), "System Competition and Network Effect," Journal of Economic Perspectives, 8(2): 93-115.

Larson, A.C. and Lehman, D.E. (1997), "Essentiality, Efficiency, and the Efficient Component-Pricing Rule," Journal of Regulatory Economics, 12, 71-80.

Larson, A.C. (1998), "The Efficiency of the Efficient-Component-Pricing Rule: A Comment," The Antitrust Bulletin, 43, 403-428.

Mamada R. and Perrings C. (2020), "The effect of emission charges on output and emissions in dynamic Cournot duopoly," Economic Analysis and Policy, 66, 370-80.

Matsushima, N. and Mizuno, T. (2012). "Profit-enhancing competitive pressure in vertically related industries," Journal of the Japanese and International Economies, 26, 142-152.

Matsushima, N. and Mizuno, T. (2013). "Vertical separation as a defense against strong suppliers," European Journal of Operational Research, 228(1): 208-216.

Matutes, C. and Regibeau, P.M. (1996), "A selective review of the economics of standardization. Entry deterrence, technological progress and international competition," European Journal of Political Economy, 12, 183-209.

Naimzada, A. and Pireddu, M. (2024), "Comparative dynamics analysis of the environmental policy efficacy in a nonlinear Cournot duopoly with differentiated goods and emission charges," Communications in Nonlinear Science and Numerical Simulation, DOI:10.1016/j.cnsns.2024.107867.

Quint, D. (2014), "Imperfect competition with complements and substitutes," Journal of Economic Theory, 152(2014): 266-290.

Sonnenschein, H. (1968) "The Dual of Duopoly Is Complementary Monopoly: or, Two of Cournot's Theories Are One", Journal of Political Economy, 76(2): 316-8.

Vives, X. (2018), "Strategic complementarities in oligopoly," Chapters, in: Luis C. Corchón & Marco A. Marini (ed.), Handbook of Game Theory and Industrial Organization, Volume I, chapter 2, pages 9-39, Edward Elgar Publishing.

Willig, R.D. (1979), "The Theory of Network Access Pricing", in H. M. Trebing, (ed.), Issues in Public Utility Regulation, East Lansing Mich.: Michigan State University Public Utilities Papers.