

Dynamic Stochastic Model of Allometric Equations and Cumulative Distribution for Bio-Mass-Carbon in *Abies Religiosa* (Kunth) Schltl. Y Cham., Facing Climate Change

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Abstract

To develop a dynamic and stochastic model that incorporates verified bio-spheric interactions, in order to calculate allometric equations for the total volumetric growth of *Abies religiosa* (Kunth) Schltl. and Cham. and to determine the cumulative distribution of its biomass. This model will be applied to populations of the species in six Mexican states: Mexico, Guerrero, Hidalgo, Michoacán, Oaxaca and Puebla. The study took into account the effects of climate change on these variables. The methodological approach integrated various tools and data sources: SiBiFor numerical databases were used, incorporating NASA Power climate data and applying Ordinary Least Squares statistical models. The Random Forest package was used for predictive analysis and the Ridge Model was implemented with regression techniques, developing algorithms using the R programming language. For volumetric estimation, Newton's equations were applied. The results of the study yielded several important conclusions: allometric equations were developed to estimate the total tree volume in 2023. Linear regression models proved to be particularly relevant in this process. The validity and usefulness of the variables selected for the study were confirmed. The study highlights the importance of understanding and assessing the carbon al-storage capacity of forests, especially in the context of climate change. It also highlights the usefulness of linear regression models and variable validation for estimating carbon sequestration in forests of *Abies religiosa* (Kunth) Schltl. and Cham.

Keywords: *Allometry, Abies Religiosa, Climate Change, Forest Biomass, Stochastic Models.*

Introduction

Strengthening the ability to create mathematical models, both in theory and in practice, based on the connection between mathematical reasoning and engineering reasoning, is fundamental to address the challenges inherent to the profession [1]. A stochastic model is a quantitative representation of a real process involving random elements. These models are used to calculate the probabilities of different out-comes, given certain initial conditions [2]. Based on the work of [3], it is possible to use these models to evaluate the sensitivity of forest stands to climate change and their role as carbon reservoirs.

The tolerance ranges of plants to factors such as temperature, humidity and light radiation are narrow. Exceeding these limits can trigger alterations in their geographic distribution, with high mountain species being particularly vulnerable [4]. There are numerous investigations ranging from climate change to tree genetics to soil carbon measurement and the use of advanced mathematical models. However, the complexity of these natural systems makes it difficult to obtain a complete picture [5], despite their importance, these aspects are addressed in a reduced number of forestry studies, and their presence in Mexican research is even scarcer [6].

The objective of this work is to build a mathematical model that simulates the growth of trees in a forest, considering variables such as size, age and environmental conditions. This model will allow estimating the

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amount of carbon stored in the biomass and evaluate how this amount changes over time. In addition, it seeks to understand how factors such as climate change affect the capacity of forests to capture carbon [7-8] and highlights the relevance of carrying out more comprehensive studies in this area [9-10-11]

Materials and Methods

A study was conducted in the forests of *Abies religiosa* (Kunth) Schltdl. and Cham. present in the states of Mexico, Puebla, Guerrero and Hidalgo, using as a source of information the generic databases of the SiBiFor project [12] taking as a reference the period between 2013 and 2015 and using the methodology of the NASA POWER project [13], dendrometric data (diameter, height) were analyzed and a dynamic stochastic model was used to estimate carbon sequestration. The model was validated using statistical techniques such as linear regression and ordinary least squares [14-15]. Five exploratory techniques of the graphical method were used in this research, since it analyzes the scatter plots of the squared residuals of the model against the values of the variables, both endogenous and exogenous, in search of systematic patterns that could reveal heteroscedasticity. In case the residuals are homoscedastic, the scatter plot would take the form of an almost constant horizontal band. If patterns are observed in the graphs, there would be evidence of dependence between the squared residuals and the variables, moreover, it would be known which variable is the cause of such dependence. The graphical methods used were residuals plot, residuals versus Fitted plot, normal Q-Q plot, Scale Location Model plot and Residuals versus Lverage plot. Since Ridge Regression is a biased estimation starting from the solution of Least Squares (LS) regression, it is vital to establish the conditions for which the student's t central distribution used in LS hypothesis testing is also applicable to RR regression. The test of this important result is presented in this research. The method of allows detecting multicollinearity within a regression model of the type $Y = X\beta + \epsilon$, is used to work with models that present bias. Random Forest is a machine learning technique that groups multiple decision trees, each built on a random sample of the training data and considering a random subset of the features. This combination of trees, known as random forest, decreases the risk of overfitting and improves the overall predictive ability of the model [16] cited by [17]. Random Forest has proven to be a versatile tool in several practical applications, demonstrating its effectiveness in solving a wide range of problems. In this study, dendrometric data (normal diameter, total height and merchantable height) served as the basis for developing allometric models and cumulative distributions, allowing estimation of carbon sequestration in *Abies religiosa* (Kunth) Schltdl. and Cham forests. In addition, timber volume estimation techniques, such as the ordinary least squares method and Ridge regression [18] were used to determine the amount of carbon that forests can store, a statistical analysis was carried out. Statistical and graphical techniques were used to validate the proposed model. All these operations were performed using R statistical software and the randomforest package [19-20-21] to apply statistical and data analysis techniques and obtain relevant results.

Results

Using climate data provided by the NASA POWER project (NASA, 2021; POWER 2021) specifically solar radiation, temperature and humidity, were identified as key factors for the growth of *Abies religiosa* (Kunth) Schltdl. and Cham. These data were used to develop allometric equations that estimate the total volumetric increment and biomass distribution in these forests. In addition, they allowed us to evaluate the potential of these ecosystems to sequester carbon. The allometric equations obtained using ordinary least squares (OLS) and SiBiFor methodology are presented in Table 1-6 [22] for *Abies religiosa* (Kunth) Schltdl. and Cham. forests in the states of Mexico, Puebla, Guerrero and Hidalgo.

Table 1. Summary Of Allometric Equations Estimated by OLS With R Algorithm In 2023 for *Abies Religiosa* (Kunth) Schltdl. Y Cham. For States De México And Puebla.

Evaluation of allometric equations estimated by mathematical methods that estimate TTV with bark in <i>Abies religiosa</i> (Kunth) Schltdl. y Cham.			
Name	Equation	Adjusted (R ²)	p-value

Traditional mathematical equations adjusted by mathematical models to estimate TTV volume with cortex (m ³)			
Newton	estado de México: Vol. = $e^{-0.169430} \text{nor_dia}^{0.290672} \text{tot_heig}^{0.040909}$	0.9617	< 2.2e-16
	Puebla: Vol. = $e^{-0.232} \text{nor_dia}^{0.296} \text{tot_heig}^{0.046}$	0.9603	< 2.2e-16
Theoretical (wood logs)	estado de México: Vol. = $e^{0.79830} \text{nor_dia}^{0.06833} \text{tot_heig}^{0.12213}$	0.2733	0.1619
	Puebla: Vol. = $e^{0.022} \text{nor_dia}^{0.186} \text{tot_heig}^{0.193}$	0.9622	< 2.2e-16
Allometric equation used by government institutions by Federal Entity (state)	estado de México: Vol. = $e^{-10.49738} \text{nor_dia}^{1.52797} \text{tot_heig}^{1.50979}$	0.9739	< 2.2e-16
	Puebla: Vol = $e^{-10.1902} \text{nor_dia}^{0.7712} \text{tot_heig}^{0.5131}$	0.9743	< 2.2e-16
Allometric equations estimated by SiBiFor [20]			
México	T7_U1505_Mex: 0.00007 * Potencia(Diam, 1.87429) * Potencia(Alt, 0.96011)	0.9733	0.0
	T7_U1507_Mex: 0.00006 * Potencia(Diam, 1.70925) * Potencia(Alt, 1.13826)	0.9625	0.0
	T6_U1508_Mex: 0.00008 * Potencia(Diam, 1.72245) * Potencia(Alt, 1.06962)	0.9673	0.0
	T3_U1509_Mex: 0.00006 * Potencia(Diam, 1.73205) * Potencia(Alt, 1.14501)	0.9591	0.0
	T5_U1510_Mex: 0.00012 * Potencia(Diam, 1.94146) * Potencia(Alt, 0.67712)	0.9614	0.0
Puebla	T1_U2101_Pue: 2107: 0.00007 * Potencia(Diam, 1.7611) * Potencia(Alt, 1.02855)	0.9734	0.0
	T1_U2105_Pue: 0.00004 * Potencia(Diam, 1.89329) * Potencia(Alt, 1.04186)	0.9809	0.0
	T1_U2108_Pue: -: 0.00005 * Potencia(Diam, 1.4906) * Potencia(Alt, 1.50494)	0.9724	0.0

Table 2. Summary of Allometric Equations Estimated by OLS With R Algorithm In 2023 for *Abies Religiosa* (Kunth) Schltdl. Y Cham. for States of Hidalgo and Guerrero.

Evaluation of allometric equations estimated by mathematical methods that estimate TTV with bark in <i>Abies religiosa</i> (Kunth) Schltdl. y Cham.			
Name	Equation	Adjusted (R ²)	p-value
Traditional mathematical equations adjusted by mathematical models to estimate TTV volume with cortex (m ³)			
Newton	Hidalgo: Vol. = $e^{-0.2147} \text{nor_dia}^{0.2576} \text{tot_heig}^{0.0872}$	0.9461	< 2.2e-16
	Guerrero: Vol. = $e^{-0.249} \text{nor_dia}^{0.292} \text{tot_heig}^{0.055}$	0.9614	< 2.2e-16
Theoretical (wood logs)	Hidalgo: Vol. = $e^{0.798} \text{nor_dia}^{0.068} \text{tot_heig}^{0.122}$	0.2733	0.1619

	Guerrero: Vol. = $e^{0.008} \text{nor_dia}^{0.193} \text{tot_heig}^{0.192}$	0.972	< 2.2e-16
Allometric equation used by government institutions by Federal Entity (state)	Hidalgo: Vol. = $e^{0.188} \text{nor_dia}^{0.164} \text{tot_heig}^{0.188}$	0.9663	< 2.2e-16
	Guerrero: Vol. = $e^{0.4298} \text{nor_dia}^{0.1999} \text{tot_heig}^{0.0518}$	0.9571	< 2.2e-16
Allometric equations estimated by SiBiFor [20]			
Hidalgo	T8_U1303_Hgo: 0.00003 * Potencia(Diam, 1.568) * Potencia(Alt, 1.47853)	0.9645	0.0
Guerrero	T14_U1203_Gro: 0.00005 * Potencia(Diam, 1.71575) * Potencia(Alt, 1.20382)	0.9793	0.0

The following table presents the allometric equations estimated by OLS (Ridge Model) for *Abies religiosa* (Kunth) Schltdl. and Cham. considering those climatic variables that are relevant. These equations were developed using the climatic data provided by the POWER project. It was found that the linear allometric equations estimated by OLS with Ridge Model provide a better volumetric approximation to climate change.

The allometric equations estimated by OLS with the Ridge Model for *Abies religiosa* (Kunth) Schltdl. and Cham. With climatic variables significant by region are “Mexico”, with

Adjusted R – squared 0.07893 , p – value < 2.2e – 16 y $\text{Vol}_{\text{Federal Entity (state)}} = 7138.0687 + 18.1048 \text{PS} - 95.7780 \text{TS} + 125.0870 \text{T2M} - 72.7695 \text{QV2M} + 12.2876 \text{RH2M} - 420.0276 \text{GWETROOT} + 0.9817 \text{CLOUD_AMT} - 261.4542 \text{TOA_SW_DWN} - 1.1038 \text{PRECTOTCORR} - 0.025 \text{ALLSKY_SFC_PAR_TOT} + 0.076 \text{CLRSKY_SFC_PAR_TOT}$; con Adjusted R – squared 0.1958, p – value < 2.2e – 16 y $\text{Vol}_{\text{Wood logs}} = 4002.02659 + 6.87207 \text{PS} - 49.98427 \text{TS} + 66.17008 \text{T2M} - 32.32558 \text{QV2M} + 5.96720 \text{RH2M} - 199.45425 \text{GWETROOT} + 0.16191 \text{CLOUD_AMT} - 139.49038 \text{TOA_SW_DWN} - 0.38913 \text{PRECTOTCORR}$;

“Puebla”, con Adjusted R – squared 0.1233 , p – value < 2.2e – 16 y $\text{Vol}_{\text{Federal Entity (state)}} = -0.0791480 + 0.0010533 \text{PS} - 0.0042929 \text{QV2M} + 0.0007720 \text{RH2M} - 0.0281312 \text{GWETROOT}$ con Adjusted R – squared 0.2127 , p – value < 2.2e – 16 y $\text{Vol}_{\text{Wood logs}} = 5.1688 + 0.3378 \text{PS} - 1.5932 \text{TS} + 4.1696 \text{QV2M} - 0.5159 \text{RH2M} - 8.2580 \text{GWETROOT}$;

Describes the term used by OLS with Ridge Model; States of Mexico, Puebla, Guerrero and Hidalgo: Indicates the states in which allometric equations were estimated; Method for commercial volume dimension: Shows the method used to calculate the commercial volume dimension of trees (Newton; Theoretical Logs and State); Percentage of variance explained (R²); Intercept: Refers to the intercept of the allometric equation, which is the value of the dependent variable when all independent variables are equal to zero; Total height (h), Normal diameter (diam) and other climatic variables: estimated coefficients for each of the independent variables included in the allometric model, indicating how each variable contributes to the calculation of tree volume.

Discussion

The numerical bases of a cut over time (2013-2015), collected in the field by the technical staff of SiBiFor for the timber species *Abies religiosa* (Kunth) Schltdl. and Cham. in the states of Mexico, Puebla, Guerrero and Hidalgo were divided into the categories “commercial volumetric dimension”, with dasometric measurements of dominant and co-dominant trees and “non-commercial volumetric dimension”, with

dasometric measurements of suppressed trees, with the objective of estimating a model for each category separately, since it would not justify its use only with the statistical criterion of R^2 and, consequently, the predictions would be biased. Based on this, the use of Newton's mathematical equation to estimate timber volumes, theoretical models for elusive dendrometric types (volume per log), allometric equations used in the field by the governmental institutions of the Federal Entities, OLS and R algorithms, the allometric equations “commercial volumetric dimension” of *Abies religiosa* (Kunth) Schltdl. and Cham. were estimated using mathematical models in the states of Mexico, Puebla, Guerrero and Hidalgo. According to the validation graphs, the models meet the evaluation criteria. Allometric equations estimated “non-commercial volumetric dimension” of *Abies religiosa* (Kunth) Schltdl. and Cham. by mathematical models in the states of Mexico, Puebla, Guerrero and Hidalgo show low ANOVA statistical criteria. This is due to the fact that the dasometric measurements of suppressed trees differ significantly. According to the validation graphs, the models do not meet the evaluation criteria. Based on the field cross-validation, the traditional mathematical equation that has the best accuracy in estimating the total tree volume with bark for *Abies religiosa* (Kunth) Schltdl. and Cham. in the states of Mexico, Puebla, Michoacán, Guerrero, Hidalgo and Oaxaca is Newton with an average evaluation of 95. The allometric equation estimated by mathematical models with the best estimation of the total tree volume with bark for this timber forest species is the allometric equation estimated by mathematical models with the best estimation of the VTA. Theoretical models for elapsed dendrometric types (volume per logs) in the states of Mexico, Puebla, Guerrero and Hidalgo with an average evaluation of 90.5%. Based on the use of the numerical bases of a time-slice (2013-2015) of SiBiFor for the timber species *Abies religiosa* (Kunth) Schltdl. and Cham, the application of the package 'randomForest' for R to interpolate and estimate the missing information of the variable “Age_no_rings” in the UMAFOR 1508 of the state of Mexico, 2105 of the state of Puebla, the use of Power data, in monthly time series format from (1980-2021), and the discernment of which climatic variables are relevant in the volumetric growth of a tree, allometric equations of “commercial volumetric dimension” were estimated for *Abies religiosa* (Kunth) Schltdl. and Cham. with significant climatic variables in the states of Mexico, Puebla, Guerrero and Hidalgo. This is because, according to [23], the advantage of Ridge regression over least squares is rooted in the bias-variance trade-off. As λ increases, the variance of the Ridge regression decreases, but the bias increases. Furthermore, in the least-squares coefficient estimates, which correspond to the Ridge regression with $\lambda=0$, the variance is high but there is no bias. Additionally, the Paterson C. P. V. Productivity Index indicates that climate is one of the essential elements in forest production, even Paterson limited himself to studying temperature, humidity, time of growing season and radiation intensity, since climate is the main ecological factor at regional scale and its influence is mainly expressed in the changes of vegetation physiognomy and floristic composition [24]. In order to evaluate the mathematical model that approximates the timber volume of *Abies religiosa* (Kunth) Schltdl. and Cham. based on linear allometric equations estimated by OLS with Ridge Model for normal diameter, height and significant climatic variables, simulations were performed with the linear coefficients of diameter-height per tree located in the UMAFOR and significant climatic variables with results that indicate a better volumetric approximation by Newton's mathematical equations and theoretical models for elusive dendrometric types (theoretical or volume per logs). According to [25]. and Cham. in Nuevo San Juan, Michoacán. *Madera y Bosques* 2001, 7, 27-47. The long-term carbon content per hectare indicates that the total unit capture reaches 217 tC/ha, in which 94 tC/ha come from the soil, 74 tC/ha from biomass and 49 tC/ha from products, where from the first turn the biomass presents a constant increase in the values of carbon content until 100 years, when it stabilizes and the carbon content in the products is until the year 2000 when it begins to stabilize”. Although, [26] Estimation of biomass and carbon in two tree species in La Sierra Nevada, Mexico. *Revista Mexicana de Ciencias Agrícolas* [27] state that the biomass distribution in the trees was 77.07% in the stem, including the stump, 8.45% in branches, 9.01% in foliage, the equation to estimate biomass was $B=0.013DN^{3.0462}$, with R^2 of 0.9909 and for the carbon biomass content $C=0.0065DN^{3.0484}$, with R^2 of 0.9914”. However, the adjusted model is of the form $Y=bX^k$ where the dependent variable (Y) is biomass or carbon, expressed in kilograms and the independent variable the normal diameter taken at 1.3 m height (DN) expressed in centimeters determines that “the trees of this species concentrate most of the aerial carbon in the stem followed by the branches and finally the foliage”. Based on the above, this research provides valuable information on the growth and yield of *Abies religiosa* (Kunth) Schltdl. and Cham. forest in the states of Mexico, Puebla, Guerrero and Hidalgo, response to different environmental factors through the development of its allometric equations, as well as the

evaluation of carbon sequestration, as they are fundamental to understand the dynamics of the forest ecosystem in the face of climate change. The findings of this study are in agreement with previous research that has highlighted the importance of understanding forest responses to environmental changes [28], the need for validated stochastic dynamic models to estimate allometric equations and assess carbon sequestration [29], the application of the principles of parsimony and simplicity in model selection [30]. However, it has certain limitations, such as being focused on the states of Mexico, Puebla, Guerrero and Hidalgo; therefore, the results may not be generalizable to other forest types. In addition, the network of sampling sites in Mexico is not yet sufficient to derive complete conclusions about the growth of many species in stands under different treatments and in sites with different productivity [31-32]. Finally, forestry research in Mexico still faces challenges in terms of data availability and development of models that are widely used in practice [33-34] and that provide a solid basis for future research and the development of forest management strategies based on mathematical evidence [35].

Conclusions

Based on the use of the SiBiFor databases, the application of OLS, R algorithms, estimated ANOVAS, F-distribution theorems with p degrees of freedom in the numerator and $n-p-1$ in the denominator, p -value, interpretative criteria of frequentist statistics, residual validation plots, residuals versus fitted, normal Q-Q, scale location model, residuals versus leverage, α , use of Power data from the NASA methodology, application of OLS, package random Forest, Ridge Model with its regression and the R2 statistical criterion, simulation of the linear allometric equations estimated by OLS with Ridge Model and the significant climatic variables, H_0 is rejected and it is accepted that the linear regression models are significant and explain the variability of Y ; allometric equations of “commercial volumetric dimension” were estimated for *Abies religiosa* (Kunth) Schltdl. and Cham. with significant climatic variables; the traditional mathematical equations and allometric equations estimated with the best precision in the estimation of total tree volume with bark in the states of Mexico, Puebla, Michoacán, Guerrero, Hidalgo and Oaxaca is Newton and, in second place, theoretical models for elusive dendrometric types (volume per log); the allometric equation estimated by mathematical models with the best estimation of the VTA cc is theoretical models for dendrometric excurrent types (volume per log) in the states of Mexico, Puebla, Michoacán, Guerrero, Hidalgo and Oaxaca with an evaluation of 90.5%; there is a better approximation of volumetric versus climatic change by Newton's mathematical equations and theoretical models for elusive dendrometric types (volume per log).

Patents

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