

# Fuzzy LMAW Method for Digital Marketing Technologies Assessment in Industry 4.0 age: Saudi Food Company Study Case

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## Abstract

*Industry 4.0 is a new technological era which is revolutionizing business operations and customer interactions. Labor markets and policymakers entrusted with promoting pertinent skills and employment opportunities face significant challenges because of the rapidly changing technological landscape which is marked by automation and irreversible changes in the labor market. In this study, the Fuzzy Logarithm Methodology of Additive Weights (FLMAW) is used to evaluate Digital Marketing Technologies (DMT) in a real-world example from a Saudi food company. The new fuzzy MCDM model, which planners and decision-makers can use when assessing and selecting DMT in I4.0. Furthermore, a case study is carried out to illustrate the feasibility of the suggested approach. The results of the FTOPSIS and FCOPRAS approaches were compared with those of FLMAW to show which was more effective. In the second stage, the rank reversal problem was used to compare the sensitivity of the FLMAW method with the FTOPSIS method. As a result, the FLMAW method produces accurate outcomes.*

**Keywords:** I4.0, Technology, Digital Marketing, Fuzzy set, Logarithm Methodology of Additive Weights, MCDM, Rank reversal problem.

## Introduction

In line with Saudi Arabia's Vision 2030, the Kingdom has embarked in recent years on a major transformational drive toward social progress, economic diversification, and technological advancement (Alnasser et al, 2024). One of the main tenets of this Vision is digital transformation, which seeks to give people a range of opportunities in the digital era to attain a higher standard of living and economic sustainability (Alotaibi, 2021). Considering this importance, Saudi Arabian enterprises are rethinking their business models and embracing digital technologies more and more to stay ahead of the constantly changing global scene (Alnasser et al, 2024). The most recent annual report from the Saudi Arabian Committee for Digital Transformation states that digitalization is crucial to creating a dynamic society and a booming economy. Like that, digitalization has transformed Saudi businesses' value chains completely, particularly in how they market and distribute their products. The Kingdom has made significant progress in its digital transformation in response to the need to abandon conventional practices in the face of a rapidly evolving digital landscape and an unparalleled surge in the use of remote platforms.

The Saudi government thinks adopting the Fourth Industrial Revolution (namely I4.0), based on digitalization, is essential for the country's successful sustainable development. The process of integrating technology throughout an industrial organization to fundamentally alter how it functions internally and markets its goods or services externally is known as "digital transformation."

According to Sterev (2017), industry 4.0 is a modern business culture that is formed by a combination of innovations, human capital, and a fresh entrepreneurial mindset. Consequently, Industry 4.0 has resulted in increased production process flexibility and a focus on customer needs and demands to overcome the complexity of the market. As with the other production processes, these elements of Industry 4.0 have a big influence on the marketing process, and they've caused it to change to make sure that the companies use the latest technology that matches the demands of the customer's needs.

In recent years with the occurrence of the industry 4.0 revolution, digital marketing has become an increasingly popular option for companies who wish to market their goods and services to a wider and more varied audience. Digital marketing (DM) refers to the promotion of goods and services using digital

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media, mainly computers and mobile phones. Any marketing media that is delivered electronically is considered digital marketing.

Marketing is a turbulent, changing, and dynamic business activity, and its role has changed significantly in recent years due to various crises, including the effects of rapid technological changes in some industries (Bala and Verma, 2018). The new lifestyle and the migration of many customers into the virtual world have led to a change in customers' purchasing habits. The marketing strategy used by organizations among them is digital marketing.

Digital marketing (DM) refers to promoting goods and services using digital media, mainly computers and mobile phones. Any marketing media that is delivered electronically is considered digital marketing (Al-Harazah, et al, 2020). This means that it comes in many forms, with the two main categories being online and offline marketing, and it can be considered a branch of traditional marketing, as it makes use of similar concepts in the marketing process (Buratti, 2018). DM is a form of marketing where the marketing activity is carried out through channels such as search engines, websites, social media, email, and mobile apps. It's paramount to note that it's different from traditional marketing since digital marketing is constantly changing.

When formulating digital marketing strategies, managers must consider numerous alternatives and multiple criteria. This often needs to be done in a group environment and is generally a complex and unstructured process. Traditionally, the decision would be made using methods such as ranking or trial and error, with techniques such as SWOT analysis being used to evaluate alternative strategies. This is insufficient for complex decisions with multiple alternatives and conflicting objectives.

Decision-making is the selective process of finding a decision, gathering information, and evaluating alternative outcomes (Sriram et al, 2022). Multi-criteria decision-making (MCDM) is an effective tool used to aid in problematic decisions by considering multiple criteria that depend on conflicting objectives.

Multi-criteria decision-making is an analysis aimed at the selection of an option or of different interventions, identified as independent actions for a given attribute or set of attributes, evaluated based on a well-defined criterion function, and providing a recommendation (or act) on the best option concerning the criteria. In simple words, it is a process of evaluating and making decisions between different alternatives to achieve an optimal solution to a problem. A bad decision in marketing can affect and lead to big losses and failure in reaching the aim. Digital marketing involves various decisions inside the complex world of the internet.

There has been a lot of research studying the application of the MCDM method for evaluating digital marketing technologies. The contribution of the current study will be as the first study using the Logarithm Methodology of Additive Weights in Fuzzy Environments to evaluate the DM technologies in Industry 4.0. The main problematic questions in this research are: What are the DM technologies in Industry 4.0 cited in a literature review? Which is the best among them? What are the key criteria when selecting a DMT in I4.0? How can we evaluate and select it? What are the best MCDM methods that can be used for selecting DMT in Industry 4.0 compared to the other methods?

The contribution of the present study will be as the first paper studying digital marketing technology in a fuzzy environment by considering the Fourth Industrial Revolution (I4.0). Moreover, the evaluation and selection of a DMT considered industry 4.0 using the FLMAW method has not been studied in the literature up to today. Also, this paper studies this subject in a real study case in Saudi Arabia, which is the first research on the topic, as it had not been addressed previously in the Kingdom.

This study's main contribution can be summarized as follows:

- As previously stated, the fuzzy evaluation of digital marketing technology in the Fourth Industrial Revolution has not been addressed in previous research and this research is considered the first to address this topic.

- Illuminate the main DM technology and criteria associated with the era of I4.0 by the fuzzy data.
- Proposed the fuzzy LMAW for identifying the different criteria for selecting and evaluating the DMT.
- Evaluate and select the appropriate digital marketing technology in the real case in Saudi Arabia.
- Compared the fuzzy LMAW method results with the Fuzzy TOPSIS and Fuzzy COPRAS methods to demonstrate if the final rank of DMT is the same or different.
- Applied the rank reversal problem to compare the sensitivity of the FLMAW method with the FTOPSIS method to demonstrate the feasibility of the proposed methodology.

The main goal of this paper article is to provide a thorough decision-making process for selecting the right DM technology in I4.0, considered a fuzzy environment. The FLMAW method will be used because it has multiple benefits compared to other MCDM methods. A new multi-criteria decision-making framework with a methodology for figuring out the weight coefficients of the criteria is presented by the FLMAW method. Like the FTOPSIS method, the FLMAW method also defines the fuzzy distance between alternatives and reference points, utilizing similar underlying principles. When the initial decision-making matrix's number of alternatives was altered, the LMAW approach demonstrated superior stability and robustness of results in comparison to the FTOPSIS method.

One benefit of the FLMAW method was that it did not result in rank reversal issues. Consequently, the FLMAW approach demonstrated a high degree of stability and dependability of the outcomes in a dynamic setting, as well as stability when handling bigger data sets. (Pamučar et al 2021). This has also been confirmed by the case study discussed in this document.

The following part of the paper is divided into three sections: the second section is a review of pertinent literature on MCDM methods and digital marketing technologies (DMT) in I4.0. The fuzzy LMAW methodology is explained in the third section. The FLMAW approach is used in a real-world case with a thorough computation provided, and the criteria that determine the DMT selection are presented. The fourth section of the paper involves validating and discussing the results obtained by the FLMAW method, through a comparison with other MCDM methods and an application of the rank reversal problem to measure the robustness of the results obtained by the FLMAW method.

## Literature Review

Digital marketing is the marketing of products or services using digital technologies, mainly on the Internet, but includes mobile phones, display advertising, and any other digital medium (Desai 2019). In literature, there are technologies used in digital marketing in Industry 4.0 which are:

**Internet of Things (IoT):** Due to the Internet of Things (IoT), businesses can benefit from real-time marketing and accelerated marketing processes. The Internet of Things made it possible to adjust the marketing strategies and it has been spent less time on marketing and sales, and the sales process has become more streamlined and practical (Akhai and Khang (2024), Abdullah et al 2023, Rosário and Dias 2022, Trung and Thanh 2022).

**Cloud manufacturing (CM):** The cloud manufacturing (CM) model is a new type of network manufacturing that uses the internet and CM service platforms to arrange online manufacturing resources or the manufacturing cloud, based on user requirements and offers a range of on-demand manufacturing services Guo et al (2024).

A web-based management dashboard, cloud-based collaboration, and cloud-based software are all provided to manufacturers through cloud-based manufacturing. Data storage, analytics, architecture, and design are all made possible by cloud computing services, which are inexpensive and available from any location using current computer hardware. As a result, cloud computing helps to build the essential digital infrastructure

required for successful marketing. Digital marketers, for example, can access data remotely in a variety of file formats. It follows that time, and resources are minimized in the process of managing and maintaining efficient marketing (Rosário and Dias 2022). In the manufacturing sector, CM has grown in popularity as a research topic such as Guo et al (2024), Liu et al (2024), Ellwein et al (2023), Delaram(2021).

**Big Data Analytics and Customer Profiling:** Fan et al. (2015) defined Big Data as “the amount of data just beyond technology’s capability to store, manage and process efficiently”. The field of "big data" studies techniques for systematically analyzing, extracting information, and handling data volumes that are too large or complex which the standard data-processing application software to handle.

Big Data marketing is associated with database marketing, which is the process of gathering, combining, and selling personal information. There are more ways to collect data for targeted ads thanks to the growing popularity of social networking sites like Facebook, Instagram, and Twitter, as well as e-commerce sites like Amazon and Alibaba. Big Data applications allow businesses to reach out to consumers more easily and respond to their inquiries, giving them a better understanding of their "Insight Rosário and Dias (2022), Itani et al (2024), Mullins (2021).

**Augmented/Virtual Reality (AR):** Marketers have rapidly become interested in Augmented Reality due to its potential to revolutionize the customer experience, from product discovery to purchase decisions (Trung and Thanh (2022)). By enabling interaction with online features, augmented reality applications give the catalog of online retailers a new dimension. Using an augmented reality application, you can project the product into the desired three-dimensional space and view it from any angle. Put differently, augmented reality (AR) gives customers a smoother, more convenient way to shop by enhancing the visual representation of products in catalogs and online stores (Trung and Thanh (2022), (Zhou et al (2024)).

**Artificial Intelligence (AI):** A branch of computer science called artificial intelligence (AI) seeks to enable computers to automate intelligent behaviors akin to those of humans. As technology has developed, artificial intelligence has been applied in several industries, including marketing, labor, health care, security, and transportation. The use of AI in marketing is related to studying consumer behavior, gathering client data, and responding to inquiries about the products and services offered by the business. Artificial intelligence will play a major role in shaping the future of digital marketing and it will have new opportunities thanks to the power of AI. (Trung and Thanh (2022)

In 2021 Vlacic et al. defined marketing AI as creating artificial agents that use data about customers, focal companies, and competitors to take or suggest marketing actions to attain the best results. AI marketing involves leveraging intelligence technologies to gather and analyze customer data to gain critical insights, anticipate their behaviors and activities, and make automated decisions about marketing initiatives and progress. nowadays several trends in scientific research in AI in different fields e.g. Akhai et al (2024), Akhai (2024), Akhai and Kumar (2024), and Akhai (2023).

### 3D and 4D Printing

Tridimensional (3D) and four-dimensional (4D) printing are related to the process of creating objects by carefully layering materials until the intended structure is achieved.

The distinctive dimensions of the final product distinguish 3D printing from 4D printing. 4D printing technology adds time as a new dimension compared to the three dimensions (length, width, and height) of 3D printing offers. These technologies are intricate and require several steps, starting with material selection and ending with the printing procedure. The benefits of 3D and 4D printing include short lead times, comparatively low costs, and the ability to customize products. These technologies are also very sophisticated and innovative. Tridimensional (3D) and four-dimensional (4D) broaden the scope of product personalization, provide the highest degree of customization, and allow customers to participate in quick prototyping and testing of products that are already in the research and development stage. Nosalska and Mazurek (2019), Nasr et al (2024), Pereira et al (2024).

**Modeling and Simulation (MS):** The MS enables the creation of a digital twin, providing an opportunity to test a product's functionality before purchasing it (Nosalska and Mazurek (2019).

**Cybersecurity (CS):** is a crucial element of digital marketing since CS protects customer trust and brand reputation. Cybersecurity protects an internet- and technology-driven digital marketing environment from potential threats. A breach that exposes personal information about a customer can seriously jeopardize their privacy and cause financial and brand losses. Businesses must safeguard the private information of their clients because identity theft and personal data breaches are two of the most prevalent risks in the digital world (Khatun et al (2023), Alqurashi and Ahmad I.(2024)).

The digital marketing industry is vulnerable to cyberattacks, because of its complex network of online tools and platforms. A compromised online presence or hacked website can disrupt business operations, undermine trust, and damage a brand's reputation.

**Blockchain:** is a decentralized, digital technology rapidly expanding and leaving its mark on the field of DM. The term "blockchain" refers to distributed ledger technology (DLT), which has revolutionized technology in recent years. DLT uses cryptographic hashing and decentralization to make any computerized resource's history unchangeable and simple. Blockchain is revolutionizing DM by taking away companies' rights to collect consumer data without compensating them for its worth. Marketers can take advantage of the technology's transparency and data protection features, which are desirable qualities that consumers look for in today's digital environment. Blockchain is one of DM's most significant topics because of its potential applications in the industry (Bansal et al (2021), Yang et al (2024), Yuan and Wu (2024)). In the other hand, Machine Learning is a subset of AI that focuses on developing algorithms that enable machines to learn and make predictions or decisions based on data without being explicitly programmed (Chaitanya et al, 2023).

The rapid evolution of Industry 4.0 technologies gave businesses more options. The decision-making process for various Industry 4.0 components is a complicated problem that necessitates complex control subsystems. Most real-world problems have multiple and frequently conflicting criteria, objectives, and goals, so Multi-Criteria Decision Making (MCDM) methods are important for systematically dealing with these problems.

The interest in industry 4.0 and MCDM approaches is growing every day, and some scientific papers are presented in the literature. Krstic et al (2022) evaluated the applicability of various Industry 4.0 technologies in the reverse logistics (RL) sector to point out the most applicable ones by using hybrid BWM and COBRA methods. Zayat et al (2023) investigated a systematic literature review of multi-attribute decision-making methods and applications in Industry 4.0. This research showed the trend of interest in MADM applications within the scope of Industry 4.0 over the years. Javaid et al (2023) proposed a framework to identify and analyze the significant barriers and solutions to explore the barriers of modern technology adoption and their mitigating solutions to align with Industry 4.0 objectives. These barriers are ranked with the help of expert opinions by using the BWM method and the identified solutions are ranked using the combined compromise solution (CoCoSo) method. Aballay et al (2023) proposed a methodological proposal for technology selection in the context of Industry 4.0 manufacturing by using FANP and FAHP. Miskic et al (2023) proposed an integrated MEREC and fuzzy MARCOS to evaluate the applicability of nine I4.0 technologies was evaluated based on 15 sub-criteria within three main groups of criteria, namely, technological, social and political, and economic and operative in logistics centers. Sahoo et al (2024) presented comprehensive review paper places a significant focus on the role of multi-criteria decision-making (MCDM) methodologies and emerging trends in the context of supplier selection within Industry 4.0.

According to marketing principles, digital marketing should be done respectfully to convince marketers to understand the consumer's decision-making process and analyze the market well. By understanding the market and through good anticipation, an article can be created to fulfill consumer needs. Researchers and decision-makers in the field of DMT have come to favor multicriteria decision-making (MCDM) in recent years.

Multicriteria decision-making techniques are applied in the “digital marketing” area in several scientific research. For example, Watrobski et al. (2016) evaluated marketing management employing the TOPSIS approach. Khatwani and Das (2016) used a hybrid multicriteria decision-making method to calculate the demographic parameter role in information channels. Mukul et al (2019) proposed Hybrid AHP and COPRAS methods for the evaluation of the DMT. Trung and Thanh (2022) used Spherical Fuzzy AHP and TOPSIS to evaluate and select the digital marketing technologies procedure. Kahraman et al (2016) summarized and classified the literature on digital marketing, which uses the fuzzy multicriteria decision-making methods (MCDM) and predicts the future directions for digital marketing. Şengül and Eren (2016) Used the Fuzzy AHP-Fuzzy TOPSIS for Selecting Digital Marketing Tools. Kaltenrieder et al (2016) proposed the FANP framework to enhance the interaction between customers and marketers (i.e., involved stakeholders) in DM. Imanova and Gunel (2020) proposed a new methodology using the Z-environment for the evaluation and selection of optimal DMT. Korucuk et al (2022) focused on green approaches for ICTs in logistics companies with international transportation activities and corporate identity in Istanbul by using a Fermatean Fuzzy SWARA-COPRAS. Chen (2023) presented an assessment approach to comprehend the marketing and financial competitiveness of Taiwanese banks by applying a linguistic TOPSIS integrated with a type of social network technology to assess the competitiveness of a case bank in a Kaohsiung area.

This study aims to measure and evaluate the performance of digital marketing technologies in the food company in Saudi Arabia. The data regarding the evaluation criteria used in this study have been determined by the decision-maker marketing in the company. The weights of the criteria were obtained by the fuzzy LMAW approach, which is one of the new methods in which both qualitative and quantitative data can be used together, and the opinions of decision-makers can be included in the weighting process of the criteria. This new method can be used to select the best alternative from a set of options as well as determine weight coefficients for criteria. The advantage of this model is that the expert's opinions are included in the analysis by the LMAW method. With some modifications, it has been used in many fields to solve a variety of research problems; an analysis of the literature utilizing this method is provided in Table 1, along with details about the LMAW method applications and the combination of it with other MCDM approaches (Tešić et al 2023).

**Table1. Literature review of the LMAW Applications (source: Tešić et al 2023).**

<b>Applications</b>	<b>References</b>	<b>Methods</b>
Risk assessment for light goods vehicles on two-lane road sections	Subotic et al 2021	Rough Dombi LMAW
Logistic application	Pamucar et al 2021	LMAW
Loader selection	Bozanic et al 2021	Neuro-fuzzy systems ANFIS -LMAW
Landing operations points (LOP) in combat operations of the army	Bozanic et al 2022	Fuzzy LMAW
Evaluation of Metaverse integration of freight fluidity measurement alternatives	Deveci et al., 2022	Dombi LMAW, Dombi EDAS
risks on road sections during the transport of dangerous goods in the Serbian army	Planić, 2022	LMAW, DEA
Global Multidimensional Poverty Index	Demir, 2022	Fuzzy LMAW
Green Supplier Selection in an Uncertain Environment in Agriculture	Puška et al., 2022	Fuzzy LMAW, Fuzzy CRADIS
Evaluation of the Transitions Potential to Cyber-Physical Production System of Heavy Industries in Turkey	Görçün & Küçükönder, 2022	LMAW

Material Selection Problems	Zakeri et al., 2022	MULTiple-TRIangles ScenarioS (MUTRISS), LMAW
Performance Analysis of Regional Development Agencies	Dünder 2023	Double Normalization-based Multiple Aggregation (DNMA) method-LMAW
Selection of a Dump truck	Tešić et al 2023	Fuzzy LMAW Grey MARCOS
Sustainable technology-enhanced learning (TEL)	Štilić et al., 2023	Fuzzy LMAW
MCDM Review in Marketing	Tarnanidis et al (2025)	AHP, ELECTRE, TOPSIS, ARAS, COPRAS

## Methodology

In this research, we will apply the LMAW in a fuzzy set to determine the criteria weights of digital marketing technologies. A proposed methodology to solve a study problem is described in Fig. 1.

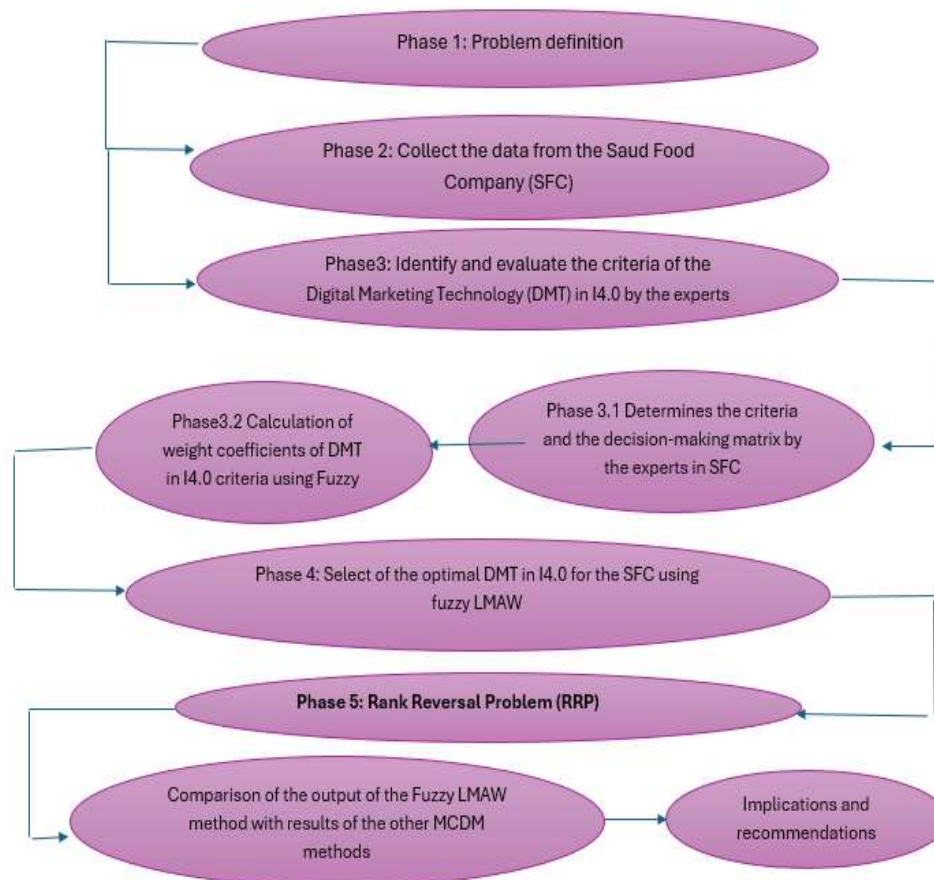


Fig 1: Flowchart of the proposed model for evaluating the DMT on Industry 4.0

## Fuzzy set theory

Zadeh (1965) developed a Fuzzy set theory (also uncertain sets theory) to address ambiguity resulting from imprecise and uncertain data. Expressing ambiguous data is recognized as one of the principal contributions of the uncertain set theory. The fuzzy theory is based on mathematical operators. A membership function (MF) that gives each object a degree of membership, a value that belongs to the interval [0,1] characterizes such a set. The triangular fuzzy number, or NFT, membership function is depicted in Fig 2.

Fuzzy LMAW definitions have been borrowed from various sources. The following is a presentation of these definitions.

**Definition 1:** A fuzzy theory  $\tilde{A}$  in a universe of discourse  $X$  is characterized by an MF  $\mu_{\tilde{A}}(x)$  that represents each element  $x$  in  $X$  to a real number between zero and one.

$\mu_{\tilde{A}}(x)$  is a function of the grade of membership of  $x$  in  $\tilde{A}$ . The neighboring value of  $\mu_{\tilde{A}}(x)$  to unity, the greater the grade of membership of  $x$  in  $\tilde{A}$ .

**Definition 2:** a triplet  $\tilde{A} = (n_1, n_2, n_3)$  represents the triangular fuzzy number. The  $\mu_{\tilde{A}}(x)$  is the membership function of a triangular fuzzy number  $\tilde{A}$ .

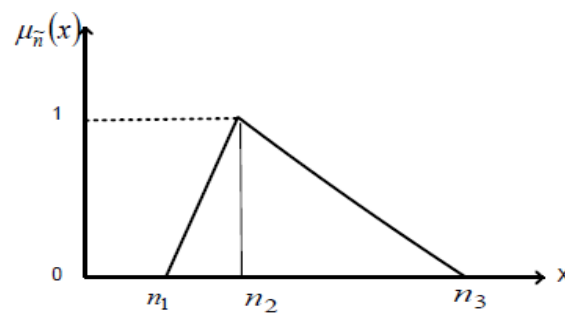


Fig.2 A triangular fuzzy number

The membership function is  $\mu_{\tilde{A}}(x)$  defined as [11]:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - n_1}{n_2 - n_1}, & \text{if } n_1 \leq x \leq n_2, \\ \frac{x - n_2}{n_3 - n_2}, & \text{if } n_2 \leq x \leq n_3, \\ 0, & \text{otherwise.} \end{cases}$$



## Fuzzy Logarithm Methodology of Additive Weights (LMAW) methodology

The relatively new Logarithm Methodology of Additive Weights (LMAW) approach was published in the literature by Pamucar et al. in 2021. It is therefore one of the newest methods for organizing options and giving different criteria different weights. The LMAW method was selected due to its ability to provide greater stability than similar techniques such as TOPSIS, its relative stability against rank reversal analysis, its mathematical framework's invariance regardless of the number of alternatives and criteria considered, its applicability to applications that combine qualitative and quantitative criteria, and its stability against rank reversal analysis. This research describes the application procedures and the calculations that were performed to arrive at the criteria weight coefficients. Based on Božanić et al 2022 the different steps of the FLMAW are presented in the following.

### Step 1: Creating expert decision-making matrices ( $\tilde{X}^e$ ).

Each expert ( $e$ ) in the set of  $k$  experts ( $1 \leq e \leq k$ ) creates an initial decision-making matrix in the first step, where he assesses  $m$  options (alternatives)  $A = \{A_1, A_2, \dots, A_m\}$  concerning  $n$  criteria  $C = \{C_1, C_2, \dots, C_n\}$ . Therefore, the matrix  $\tilde{X}^e = [\tilde{\vartheta}_{ij}^e]_{m \times n}$  is found for every expert, where  $\tilde{\vartheta}_{ij}^e$  shows a fuzzy value based that the expert  $e$  evaluated  $i$ -th alternative  $A_i$  by  $j$ -th criterion  $C_j$ . Depending on the kind of criterion, the assessment is based either on fuzzy linguistic descriptors or on quantitative indicators.

### Step 2: Creating the aggregation decision-making matrix ( $\tilde{X}$ ).

Equation (2) states that the Bonferroni aggregator is applied to aggregate the initial (expert) matrices into a single aggregate matrix.

$$\tilde{\vartheta}_{ij} = \left( \frac{1}{k(k-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^k \tilde{\vartheta}_i^{(e)p} \tilde{\vartheta}_j^{(e)q} \right) = \left\{ \left( \frac{1}{k(k-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^k \vartheta_i^{(le)p} \vartheta_j^{(le)q} \right)^{\frac{1}{p+q}}, \left( \frac{1}{k(k-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^k \vartheta_i^{(me)p} \vartheta_j^{(me)q} \right)^{\frac{1}{p+q}}, \left( \frac{1}{k(k-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^k \vartheta_i^{(re)p} \vartheta_j^{(re)q} \right)^{\frac{1}{p+q}} \right\}$$

where  $\tilde{\vartheta}_{ij}$  displays average values that were obtained through the use of the Bonferroni aggregator.;  $p, q \geq 0$  are the Bonferroni aggregator stabilization parameters,  $e$  presents the  $e$ -th expert  $1 \leq e \leq k$ ,  $l$ —the left distribution of a fuzzy number,  $r$ —the right distribution of a fuzzy number, and  $m$ — the value at which a fuzzy number's membership function equals one. The quantification of linguistic criteria is done before the aggregation.

### Step 3: Normalization of the first decision-making matrix's components.

The normalized matrix  $\tilde{X} = [\tilde{\vartheta}'_{ij}]_{m \times n}$  is found by using the Equation (3):

$$\tilde{\vartheta}'_{ij} = \begin{cases} 1 + \frac{\tilde{\vartheta}_{ij}}{\tilde{\vartheta}_j^+} = \left(1 + \frac{\vartheta_{ij}^l}{\vartheta_j^+}, 1 + \frac{\vartheta_{ij}^m}{\vartheta_j^+}, 1 + \frac{\vartheta_{ij}^r}{\vartheta_j^+}\right) & \text{if } j \in B, \\ 1 + \frac{\tilde{\vartheta}_{ij}^-}{\tilde{\vartheta}_{ij}^-} = \left(1 + \frac{\vartheta_{ij}^-}{\vartheta_{ij}^r}, 1 + \frac{\vartheta_{ij}^-}{\vartheta_{ij}^m}, 1 + \frac{\vartheta_{ij}^-}{\vartheta_{ij}^l}\right) & \text{if } j \in C, \end{cases} \quad (3)$$

where  $\tilde{\vartheta}'_{ij}$  displays the initial decision-making matrix's normalized values, while  $\vartheta_j^+ = \max(\tilde{\vartheta}_j^r)$ , and  $\vartheta_j^- = \min(\tilde{\vartheta}_j^l)$ , and  $l$  is a fuzzy number's left distribution,  $r$  is a fuzzy number's right distribution, and  $m$  is the number at which a fuzzy number's membership function equals 1.

#### Step 4: Forming the criteria weight coefficients.

Experts  $E = \{E_1, E_2, \dots, E_k\}$  are supposed to be consulted to determine the weight coefficients of the criteria.

##### Step 4.1: Setting priorities for the criteria.

The experts rank the criteria based on the values from the predetermined fuzzy linguistic scale  $C = \{C_1, C_2, \dots, C_n\}$ . The higher value from the fuzzy linguistic scale is assigned to the criterion that has greater significance, and vice versa. Each expert defined separately the priority vectors  $\tilde{P}^e = (\tilde{\gamma}_{C_1}^e, \tilde{\gamma}_{C_2}^e, \dots, \tilde{\gamma}_{C_n}^e)$ , where  $(\tilde{\gamma}_{C_n}^e)$  displays the value that the expert  $e$  ( $1 \leq e \leq k$ ) located to criterion  $n$  using the fuzzy linguistic scale.

Step 4.2: Expressing the absolute fuzzy anti-ideal point ( $\tilde{\gamma}_{AIP}$ ). Decision makers define this value that presents a fuzzy number which is smaller than the smallest value from the set of all priority vectors.

Step 4.3: Defining fuzzy relation vector ( $\tilde{R}^e$ ). The relationship between the elements of the priority vector and the absolute anti-ideal point ( $\gamma_{AIP}$ ) is found by using Equation (4).

$$\tilde{\eta}_{Cn}^e = \left( \frac{\tilde{\gamma}_{Cn}^e}{\tilde{\gamma}_{AIP}} = \left( \frac{\gamma_{Cn}^{(l)e}}{\gamma_{AIP}^{(r)}}, \frac{\gamma_{Cn}^{(m)e}}{\gamma_{AIP}^{(m)}}, \frac{\gamma_{Cn}^{(r)e}}{\gamma_{AIP}^{(l)}} \right) \right) \quad (4)$$

The relations vector of the expert  $e$  ( $1 \leq e \leq k$ ):  $R^e = (\tilde{\eta}_{C_1}^e, \tilde{\eta}_{C_2}^e, \dots, \tilde{\eta}_{C_n}^e)$

is found by employing Equation (4),

Step 4.4: Establishing vectors of weight coefficients  $\tilde{w}_j^e = (\tilde{w}_1^e, \tilde{w}_2^e, \dots, \tilde{w}_n^e)^T$ , for each expert independently. Formula (5) is used to obtain fuzzy values of the weight coefficients of criteria for the expert  $e$  ( $1 \leq e \leq k$ ).

$$\tilde{w}_j^e = \left( \frac{\ln(\tilde{\eta}_{Cn}^e)}{\ln(\prod_{j=1}^n \tilde{\eta}_{Cn}^e)} \right) = \left( \frac{\ln(\eta_{Cn}^{(l)e})}{\ln(\prod_{j=1}^n \eta_{Cn}^{(r)e})}, \frac{\ln(\eta_{Cn}^{(m)e})}{\ln(\prod_{j=1}^n \eta_{Cn}^{(m)e})}, \frac{\ln(\eta_{Cn}^{(r)e})}{\ln(\prod_{j=1}^n \eta_{Cn}^{(l)e})} \right) \quad (5)$$

where  $\tilde{\eta}_{Cn}^e$  displays the relation vector's components  $R^e$ ,  $\tilde{\eta}_{Cn}^{(l)e}$  the fuzzy priority vector's left distribution,  $\eta_{Cn}^{(r)e}$  the fuzzy priority vector's right distribution, and  $\eta_{Cn}^{(m)e}$   $m$  the number where the fuzzy priority vector's membership function equals 1.

Step 4.5: Aggregated fuzzy vectors of weight coefficients computation  $\tilde{w}_j = (\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n)^T$ . the weight coefficients' fuzzy vector aggregates  $\tilde{w}_j = (\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n)^T$  are acquired by utilizing the Bonferroni aggregator, as stated by Formula (6):

$$\tilde{w}_j = \left( \frac{1}{k(k-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^k \tilde{w}_i^{(e)p} \tilde{w}_j^{(e)q} \right) = \left\{ \left( \frac{1}{k(k-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^k \mathcal{W}_i^{(le)p} \mathcal{W}_j^{(le)q} \right)^{\frac{1}{p+q}}, \left( \frac{1}{k(k-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^k \mathcal{W}_i^{(me)p} \mathcal{W}_j^{(me)q} \right)^{\frac{1}{p+q}}, \left( \frac{1}{k(k-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^k \mathcal{W}_i^{(re)p} \mathcal{W}_j^{(re)q} \right)^{\frac{1}{p+q}} \right\} \quad (6)$$

where  $p, q \geq 0$  are the Bonferroni aggregator stabilization parameters, while  $\tilde{w}_j^e$  presents the weight coefficients obtained based on the assessments of the e-th expert  $1 \leq e \leq k$ ,  $\mathcal{W}_j^{(le)}$  is fuzzy weight coefficient's left distribution  $\tilde{w}_j^{(e)}$ ,  $\mathcal{W}_j^{(re)}$  provides the fuzzy weight coefficient's right distribution  $\tilde{w}_j^{(e)}$ , a  $\mathcal{W}_j^{(r_e)}$  is the right value where the fuzzy weight coefficient function  $\tilde{w}_j^e$  is equal 1.

Step 4.6: Compute the final weight coefficients  $\mathcal{W}_j = (\mathcal{W}_1, \mathcal{W}_2, \dots, \mathcal{W}_n)^T$ . Formula (7) explains how defuzzification yields the final values of the weight coefficients of criteria:

$$\mathcal{W}_j = \frac{l+4m+r}{6} \quad (7)$$

**Step 5: Compute the weighted matrix (N).**

Using Formula (8), the elements of the weighted matrix  $N = [\tilde{\xi}_{ij}]_{m \times n}$  are attained.

$$\tilde{\xi}_{ij} = \frac{2\tilde{\varphi}_{ij}^{\mathcal{W}_j}}{(2 - \tilde{\varphi}_{ij})^{\mathcal{W}_j} + \tilde{\varphi}_{ij}^{\mathcal{W}_j}} = \left( \frac{2\varphi_j^{(l)\mathcal{W}_j}}{(2-\varphi_j^{(r)})^{\mathcal{W}_j} + \varphi_j^{(r)\mathcal{W}_j}}, \frac{2\varphi_j^{(m)\mathcal{W}_j}}{(2-\varphi_j^{(m)})^{\mathcal{W}_j} + \varphi_j^{(m)\mathcal{W}_j}}, \frac{2\varphi_j^{(r)\mathcal{W}_j}}{(2-\varphi_j^{(l)})^{\mathcal{W}_j} + \varphi_j^{(l)\mathcal{W}_j}} \right) \quad (8)$$

Where

$$\tilde{\varphi}_{ij} = \left( \frac{\ln(\tilde{\vartheta}'_{ij})}{\ln(\prod_{i=1}^m \tilde{\vartheta}'_{ij})} \right) = \left( \frac{\ln(\vartheta'_{ij}^{(l)})}{\ln(\prod_{i=1}^m \vartheta'_{ij}^{(r)})}, \frac{\ln(\vartheta'_{ij}^{(m)})}{\ln(\prod_{i=1}^m \vartheta'_{ij}^{(m)})}, \frac{\ln(\vartheta'_{ij}^{(r)})}{\ln(\prod_{i=1}^m \vartheta'_{ij}^{(l)})} \right) \quad (9)$$

While  $\tilde{\vartheta}'_{ij}$  presents the elements of the normalized matrix  $\tilde{X} = [\tilde{\vartheta}'_{ij}]_{m \times n}$ , while  $\mathcal{W}_j$  shows the criteria weight elements, l—the fuzzy number's left distribution, r—the fuzzy number's right distribution, and m the value in which the membership function of a fuzzy number is equal to one.

### Step 6: Compute the final rank index of alternatives ( $Q_i$ ).

The final rank of the alternatives is defined based on the value of  $Q_i$ , where the higher value of the  $Q_i$  indicate the well-rank of alternatives. The defuzzification of the value  $\tilde{Q}_i$  give the value  $Q_i$ , corresponding to the Formula (7). The value  $\tilde{Q}_i$  is obtained by using the Formula (10):

$$\tilde{Q}_i = \sum_{j=1}^n \tilde{\xi}_{ij} = \left( \sum_{j=1}^n \xi_{ij}^{(l)}, \sum_{j=1}^n \xi_{ij}^{(m)}, \sum_{j=1}^n \xi_{ij}^{(r)} \right) \quad (10)$$

where  $\tilde{\xi}_{ij}$  is the weighted elements matrix  $\tilde{N} = [\tilde{\xi}_{ij}]_{m \times n}$ , l—the fuzzy number's left distribution, r—the fuzzy number's right distribution, and m is the number at which a fuzzy number's membership function equals 1.

### DMT assessment in Saudi Food Company

This empirical case study aims to select and assess the top digital marketing technologies in a Saudi Food Company .and demonstrates the application of the suggested MCDM pattern. Managers can handle DMT issues more effectively with this approach. in this real application, two experts in the company were consulted to evaluate DMT. They are the General Manager of Information Technology and general manager of marketing, where their opinion was considered to determine the criteria evaluation, and the top digital marketing technology considered in the SFC.

In this paper, a fuzzy LMAW method was used to evaluate digital marketing technologies, including the Internet of Things (A1), Cloud manufacturing (A2), Big Data Analytics and Customer Profiling (A3), Augmented/Virtual Reality (A4), Artificial Intelligence (A5), 3D Printing (A6), Modeling and Simulation (A7), Cybersecurity (A8), and Blockchain (A9). Based on experts' opinions and literature reviews, the criteria and sub-criteria are presented in Figure 3.

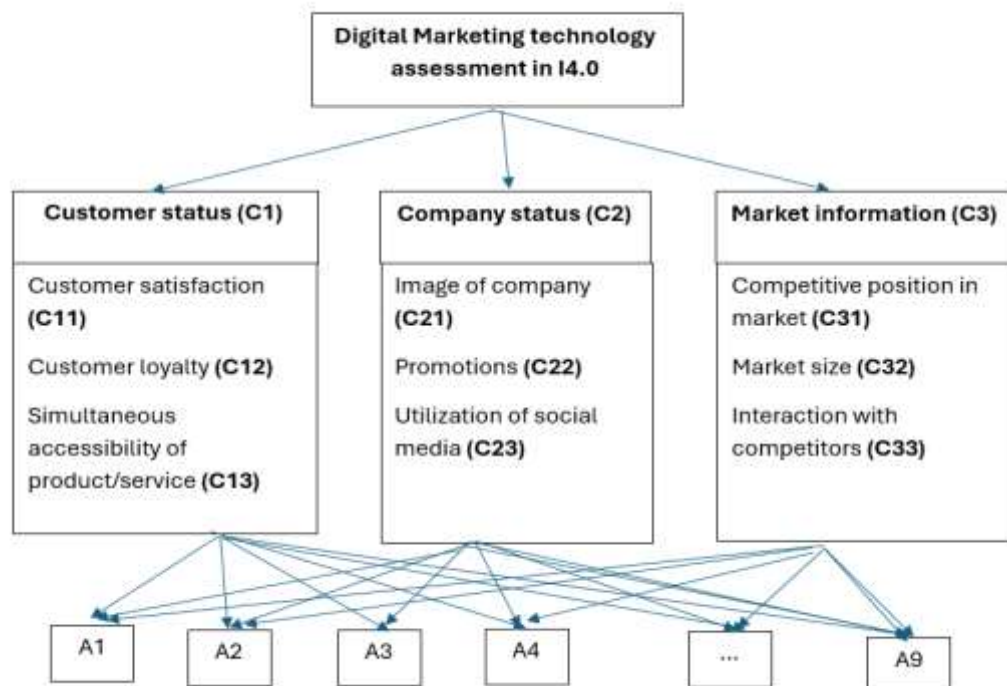


Fig 3. The criteria and sub-criteria of digital marketing technologies in I4.0

**Table 2.** Description of fuzzy linguistic descriptors.

Fuzzy Linguistic Descriptions							
Fuzzy number	(1,1,2)	(1,2,3)	(2,3,4)	(3,4,5)	(4,4,5)		
	Very small (VS)	Small (S)	Middle (M)	High (H)	Very High (VH)		

**Table 3.** Fuzzy scale for criteria prioritization.

The name of the fuzzy Linguistic Descriptor	Abbreviation	Fuzzy Number
Absolutely-low	AL	(1,1,1)
Very Low	VL	(1,1.5,2)
Low	L	(1.5,2,2.5)
Medium	M	(2,2.5,3)
Equal	E	(2.5,3,3.5)
Medium-high	MH	(3,3.5,4)
High	H	(3.5,4,4.5)
Very High	VH	(4,4.5,5)
Absolutely-high	AH	(4.5,5,5)

The experts defined the following priority vectors:

$$\tilde{p}^1 = (MH, L, VL, ML, MH, H, VH, VH, AH)$$

$$\tilde{p}^1 = (MH, E, AL, H, E, H, H, AH, AH)$$

The value of the absolute fuzzy anti-ideal point is defined as  $\tilde{Y}_{AIP} = (0.5, 0.5, 0.5)$ , and set fuzzy vectors ratio ( $\tilde{R}^e$ ) by the expression (1), as shown in Table 4.

**Table 4.** Fuzzy relation vectors ( $\tilde{R}^e$ )

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9
<b>DM1</b>	(6, 7, 8)	(3, 4, 5)	(2, 3, 4)	(4, 5, 6)	(6, 7, 8)	(7, 8, 9)	(8, 9, 10)	(8, 9, 10)	(9, 10, 10)
<b>DM2</b>	(6, 7, 8)	(5, 6, 7)	(2, 2, 2)	(7, 8, 9)	(5, 6, 7)	(7, 8, 9)	(7, 8, 9)	(9, 10, 10)	(9, 10, 10)

In the next step the vectors of weight coefficients  $\tilde{w}_j = (\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n)^T$  are determined by using the expression (2) for every expert (Table 5).

**Table 5.** Weight coefficient vectors for every expert

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9
<b>DM1</b>	(0.099, 0.116, 0.138)	(0.061, 0.083, 0.107)	(0.038, 0.066, 0.092)	(0.077, 0.096, 0.119)	(0.099, 0.116, 0.138)	(0.108, 0.124, 0.146)	(0.115, 0.131, 0.153)	(0.115, 0.131, 0.153)	(0.122, 0.137, 0.153)

<b>DM2</b>	(0.1, 0.114, 8)	(0.09, 0.105, 0.122)	(0.039, 0.041, 0.043)	(0.109, 0.122, 0.138)	(0.09, 0.105, 0.122)	(0.109, 0.122, 0.138)	(0.109, 0.122, 0.138)	(0.123, 0.135, 0.144)	(0.123, 0.135, 0.144)
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Applying the Bonferroni aggregators, by expression (3), the aggregated fuzzy weight coefficient vectors are obtained (Table 6).

**Table 6.** Aggregated fuzzy vectors of the weight coefficients of criteria.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9
<b>w<sub>j</sub></b>	(0.1, 0.115, 0.134)	(0.074, 0.093, 0.114)	(0.039, 0.052, 0.063)	(0.091, 0.108, 0.128)	(0.095, 0.110, 0.130)	(0.108, 0.123, 0.142)	(0.112, 0.126, 0.145)	(0.119, 0.133, 0.149)	(0.122, 0.136, 0.149)

Final crisp values of the weight coefficients of criteria, derived by defuzzification of fuzzy values, are obtained by using the expression (4) and shown in Table 7.

**Table 7.** Final crisp values of the weight coefficients of the criteria

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9
<b>w<sub>j</sub></b>	0.116	0.094	0.051	0.109	0.111	0.124	0.127	0.133	0.136

In the following step, the initial (aggregated) decision-making matrix ( $\bar{X}$ ) is formed, by applying the Bonferroni aggregator, as in Equation (2), and it's presented in Table 8.

**Table 8.** Aggregation decision-making matrix.

	1			1			1			1			1			1			1			1					
	0.116			0.094			0.051			0.109			0.111			0.124			0.127			0.133			0.136		
	C1			C2			C3			C4			C5			C6			C7			C8			C9		
<b>A1</b>	3.162	4.123	6.782	2.121	3.873	5.612	7.874	9.165	11.619	6.633	8.544	11.000	4.899	7.036	9.165	4.583	6.856	9.110	4.583	6.325	8.062	4.583	6.856	9.110	6.633	8.544	11.000
<b>A2</b>	2.828	4.301	6.708	8.544	9.798	12.247	7.036	8.485	10.607	7.280	9.165	11.619	4.899	7.036	9.165	3.873	6.164	8.426	6.633	8.544	11.000	3.873	6.164	8.426	7.280	9.165	11.619
<b>A3</b>	3.606	5.099	7.616	4.743	6.928	9.083	6.633	8.544	11.000	7.874	9.165	11.619	8.544	9.798	12.247	8.544	9.798	12.247	2.449	3.873	5.292	8.544	9.798	12.247	7.874	9.165	11.619
<b>A4</b>	3.873	6.164	8.426	3.000	5.000	7.000	7.211	8.544	11.000	7.036	8.485	10.607	6.083	8.544	11.000	7.280	9.165	11.619	3.873	6.164	8.426	9.381	11.358	14.526	10.247	11.662	14.577
<b>A5</b>	7.280	9.165	11.619	4.583	6.856	9.110	8.544	9.798	12.247	7.746	8.485	10.607	5.292	7.681	10.050	5.292	7.681	10.050	7.280	9.165	11.619	5.292	7.681	10.050	7.746	8.485	10.607
<b>A6</b>	3.000	5.000	7.000	8.544	9.798	12.247	7.280	9.165	11.619	6.083	8.544	11.000	6.083	8.544	11.000	7.280	9.165	11.619	3.873	6.164	8.426	8.775	11.358	14.526	6.083	8.544	11.000
<b>A7</b>	6.083	8.544	11.000	4.583	6.856	9.110	5.612	7.746	9.874	4.899	7.036	9.165	3.000	5.000	7.000	3.873	6.164	8.426	10.271	13.304	16.985	3.873	6.164	8.426	4.899	7.036	9.165
<b>A8</b>	3.606	5.099	7.616	2.828	4.301	6.708	3.162	4.123	6.782	7.036	8.485	10.607	6.083	8.544	11.000	5.292	7.681	10.050	4.000	5.745	7.483	5.292	7.681	10.050	10.075	11.662	14.577
<b>A9</b>	3.873	6.164	8.426	6.083	8.544	11.000	4.123	5.292	8.246	7.746	8.485	10.607	5.292	7.681	10.050	7.280	9.165	11.619	7.874	9.165	11.619	10.080	12.000	15.166	10.247	11.662	14.577

The normalization of the initial (aggregated) decision-making matrix, by applying Equation (3) is performed and presented in Table 9.

**Table 9.** Normalized decision-making matrix.

	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
	0.116	0.116	0.116	0.094	0.094	0.094	0.051	0.051	0.051	0.109	0.109	0.109	0.111	0.111	0.111	0.124	0.124	0.124	0.127	0.127	0.127	0.133	0.133	0.133	0.136	0.136	0.136
	C1			C2			C3			C4			C5			C6			C7			C8			C9		
A1	1.272	1.355	1.584	1.173	1.316	1.458	1.643	1.748	1.949	1.571	1.735	1.947	1.400	1.574	1.748	1.374	1.560	1.744	1.270	1.372	1.475	1.302	1.452	1.601	1.455	1.586	1.755
A2	1.243	1.370	1.577	1.698	1.800	2	1.574	1.693	1.866	1.627	1.789	2	1.400	1.574	1.748	1.316	1.503	1.688	1.391	1.503	1.648	1.255	1.406	1.556	1.499	1.629	1.797
A3	1.310	1.439	1.655	1.387	1.566	1.742	1.542	1.698	1.898	1.678	1.789	2	1.698	1.800	2	1.698	1.800	2	1.144	1.228	1.312	1.563	1.646	1.808	1.540	1.629	1.797
A4	1.333	1.531	1.725	1.245	1.408	1.572	1.589	1.698	1.898	1.606	1.730	1.913	1.497	1.698	1.898	1.594	1.748	1.949	1.228	1.363	1.496	1.619	1.749	1.958	1.703	1.800	2
A5	1.627	1.789	2	1.374	1.560	1.744	1.698	1.800	2	1.667	1.730	1.913	1.432	1.627	1.821	1.432	1.627	1.821	1.429	1.540	1.684	1.349	1.506	1.663	1.531	1.582	1.728
A6	1.258	1.430	1.602	1.698	1.800	2	1.594	1.748	1.949	1.524	1.735	1.947	1.497	1.698	1.898	1.594	1.748	1.949	1.228	1.363	1.496	1.579	1.749	1.958	1.417	1.586	1.755
A7	1.524	1.735	1.947	1.374	1.560	1.744	1.458	1.632	1.806	1.422	1.606	1.789	1.245	1.408	1.572	1.316	1.503	1.688	1.605	1.783	2	1.255	1.406	1.556	1.336	1.483	1.629
A8	1.310	1.439	1.655	1.231	1.351	1.548	1.258	1.337	1.554	1.606	1.730	1.913	1.497	1.698	1.898	1.432	1.627	1.821	1.235	1.338	1.441	1.349	1.506	1.663	1.691	1.800	2
A9	1.333	1.531	1.725	1.497	1.698	1.898	1.337	1.432	1.673	1.667	1.730	1.913	1.432	1.627	1.821	1.594	1.748	1.949	1.464	1.540	1.684	1.663	1.791	2	1.703	1.800	2

Within the weighted matrix is calculated (N) by applying the expressions (8) and (9). The values for the weighted matrix are presented in Table 10.

Table 10. Weighted decision-making matrix.

	C1			C2			C3			C4			C5			C6			C7			C8			C9		
A1	0.749	0.820	0.912	0.770	0.846	0.920	0.908	0.931	0.955	0.802	0.848	0.895	0.778	0.840	0.905	0.750	0.819	0.890	0.746	0.813	0.882	0.727	0.800	0.875	0.749	0.804	0.860
A2	0.741	0.822	0.915	0.842	0.883	0.931	0.905	0.929	0.953	0.807	0.851	0.896	0.778	0.840	0.905	0.739	0.814	0.890	0.768	0.829	0.897	0.715	0.794	0.874	0.756	0.808	0.862
A3	0.756	0.830	0.917	0.812	0.869	0.928	0.903	0.929	0.955	0.812	0.851	0.893	0.811	0.855	0.909	0.791	0.837	0.893	0.706	0.785	0.866	0.771	0.820	0.880	0.762	0.808	0.859
A4	0.759	0.840	0.922	0.788	0.856	0.925	0.906	0.929	0.954	0.807	0.847	0.890	0.789	0.849	0.912	0.780	0.834	0.894	0.731	0.811	0.893	0.774	0.828	0.891	0.778	0.821	0.869
A5	0.798	0.859	0.926	0.809	0.869	0.930	0.910	0.932	0.955	0.813	0.847	0.887	0.781	0.844	0.910	0.759	0.825	0.893	0.775	0.833	0.899	0.736	0.807	0.879	0.763	0.804	0.851
A6	0.744	0.830	0.916	0.842	0.883	0.931	0.905	0.931	0.956	0.795	0.848	0.897	0.789	0.849	0.912	0.780	0.834	0.894	0.731	0.811	0.893	0.768	0.828	0.894	0.742	0.804	0.863
A7	0.786	0.855	0.928	0.809	0.869	0.930	0.898	0.927	0.954	0.785	0.839	0.891	0.749	0.824	0.900	0.739	0.814	0.890	0.792	0.853	0.919	0.715	0.794	0.874	0.729	0.793	0.854
A8	0.756	0.830	0.917	0.785	0.850	0.923	0.881	0.913	0.951	0.807	0.847	0.890	0.789	0.849	0.912	0.759	0.825	0.893	0.737	0.807	0.881	0.736	0.807	0.879	0.776	0.821	0.870
A9	0.759	0.840	0.922	0.823	0.877	0.934	0.888	0.919	0.953	0.813	0.847	0.887	0.781	0.844	0.910	0.780	0.834	0.894	0.781	0.833	0.896	0.778	0.831	0.893	0.778	0.821	0.869

Other obtained values of the final index for ranking alternatives and the rank of alternatives are provided in Table 11.

Table 11. Rank of DMT

	$\bar{Q}_i$			$Q_i$	Rank
A1	0.749	0.820	0.912	0.823	9
A2	0.741	0.822	0.915	0.824	8
A3	0.756	0.830	0.917	0.832	5
A4	0.759	0.840	0.922	0.840	3
A5	<b>0.798</b>	<b>0.859</b>	<b>0.926</b>	<b>0.860</b>	<b>1</b>
A6	0.744	0.830	0.916	0.830	7
A7	0.786	0.855	0.928	0.856	2
A8	0.756	0.830	0.917	0.832	5
A9	0.759	0.840	0.922	0.840	3

Artificial intelligence (AI) has emerged as the most suitable digital marketing technology among nine alternatives, with a final score value of 0.860, which was found to be the best based on the final index  $Q_i$  that was obtained.

## Validation and discussion of Results

### Comparison of the results with FTOPSIS and FCOPRAS methods

In this section, the fuzzy LMAW method results are compared with those of Fuzzy TOPSIS and Fuzzy COPRAS (COmplex PROportional ASsessment). The initial data from the decision-making matrix and the weights assigned to each criterion are the same for all multi-criteria approaches. Several studies have demonstrated that the use of various models for data normalization may impact how ranking results change. For this reason, multi-criteria methods that employ various approaches to data normalization are chosen for this analysis. The results of the MCDM approaches montre that the FLMAW ranks are different that the FTOPSIS and FCOPRAS. Fig. 4 displays the outcomes of using the MCDM techniques.

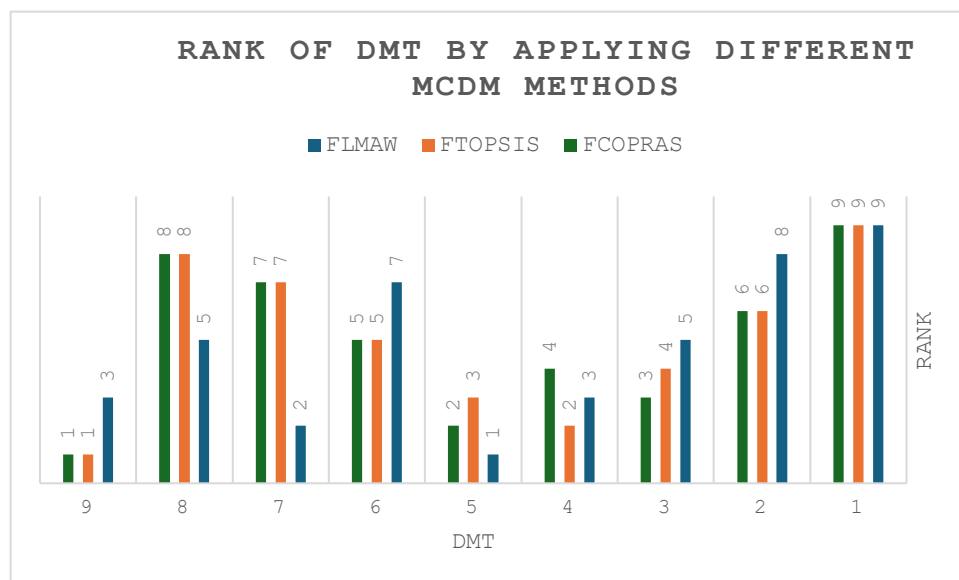


Fig 4. Comparison of the FLMAW method with other MCDM methods

### Rank Reversal Problem (RRP)

Adding new alternatives to the original cluster or removing weak alternatives from the cluster are two ways to monitor the persistence of MCDM methods. It is anticipated that in these situations, the MCDM approach won't show a notable alteration in the alternatives' order.

One of the most important issues in multi-criteria decision-making that can result in irrational and contentious choices is the rank reversal problem (RRP) (Belton 1985). The RRP is the term used to describe this phenomenon, which has received a lot of attention in literature (Pamucar et al., 2017). As a result, the next section examines how resistant the FLMAW model is to the RRP. Seven different scenarios were used to experiment. In each case, the least desirable option from the list of options under consideration was removed, and the impact of the altered number of options on the modifications to the criteria functions and ranks of the options was examined. Tables 12 and 13 present the ranks of the alternatives through seven scenarios by the application of FLMAW and the FTOPSIS for compare and demonstrate the robustness of the rank between the two methods when applying the rank reversal problem.



Table 11 makes it abundantly evident that the LMAW model produces reliable results in a dynamic setting. The outcomes demonstrated that although the RRP surfaced in the FTOPSIS method, the FLMAW method produced consistent results. Table 13 displays the FTOPSIS method application results. In the FLMAW method, all seven scenarios demonstrated stability and resistance to rank changes.

**Table 12** Ranks of DMT by scenarios - FLMAW model

	FLMAW		Scenarios						
	Weights	Rank	S1	S2	S3	S4	S5	S6	S7
A1	0.823	<b>9</b>	-	-	-	-	-	-	-
A2	0.824	<b>8</b>	<b>8</b>	-	-	-	-	-	-
A3	0.832	<b>5</b>	<b>5</b>	<b>5</b>	<b>5</b>	-	-	-	-
A4	0.840	<b>3</b>	<b>3</b>	<b>3</b>	<b>3</b>	<b>3</b>	<b>3</b>	-	-
A5	0.860	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>
A6	0.830	<b>7</b>	<b>7</b>	<b>7</b>	-	-	-	-	-
A7	0.856	<b>2</b>	<b>2</b>	<b>2</b>	<b>2</b>	<b>2</b>	<b>2</b>	<b>2</b>	<b>2</b>
A8	0.832	<b>5</b>	<b>5</b>	<b>5</b>	<b>5</b>	<b>5</b>	-	-	-
A9	0.840	<b>3</b>	<b>3</b>	<b>3</b>	<b>3</b>	<b>3</b>	<b>3</b>	<b>3</b>	-

**Table 13** Ranks of DMT by scenarios - FTOPSIS model

FTOPSIS		Scenarios						
Weights	Rank	S1	S2	S3	S4	S5	S6	S7
<b>0.511</b>	<b>9</b>	-	-	-	-	-	-	-
<b>0.570</b>	<b>6</b>	<b>6</b>	<b>5</b>	<b>6</b>	-	-	-	-
<b>0.632</b>	<b>4</b>	<b>3</b>	<b>3</b>	<b>1</b>	<b>1</b>	-	-	-
<b>0.645</b>	<b>2</b>	<b>2</b>	<b>4</b>	<b>3</b>	<b>1</b>	<b>1</b>	<b>2</b>	<b>2</b>
<b>0.643</b>	<b>3</b>	<b>5</b>	<b>2</b>	<b>4</b>	<b>3</b>	<b>2</b>	<b>3</b>	-
<b>0.615</b>	<b>5</b>	<b>4</b>	<b>6</b>	<b>5</b>	<b>4</b>	-	-	-
<b>0.559</b>	<b>7</b>	<b>8</b>	<b>7</b>	-	-	-	-	-
<b>0.548</b>	<b>8</b>	<b>7</b>	-	-	-	-	-	-
<b>0.678</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>2</b>	<b>2</b>	<b>4</b>	<b>1</b>	<b>1</b>

The analysis that has been presented leads to the conclusion that the FTOPSIS model has a rank reversal issue, which may cause illogical results to emerge in the case of variable input parameters in the initial decision-making matrix. Conclusively, the experiment presented indicates that the FLMAW approach exhibits resistance against the rank reversal problem. This research concludes that the FLMAW model supports a stable and realistic evaluation of options for resolving issues in the real world.

Therefore, after the RRP obtained the results, the experts in FCSA considered that the FLOW model provides robust and stable results compared to the other MCDM approaches.

## Conclusion

In the presented paper, triangular fuzzy numbers were used in the paper that was presented to successfully implement the fuzzy LMAW method. The actual case of the Food Company in Saudi Arabia was applied and thoroughly explained following the introduction and literature analysis.

The FLMAW method was step-by-step explained, along with basic settings about fuzzy numbers. For testing the method, specific problems about digital marketing technologies were solved, such as choosing the best option among the nine available technologies. Every step of the FLMAW method's calculation examples were given through testing. Verification of the output results was accomplished in two stages,

with testing for a particular problem confirming the FLMAW method's quality. The results were first compared with two other methods (FTOPSIS and FCOPRAS), the efficacy of which had been demonstrated through research. In the second stage, the rank reversal problem was applied to compare the method's sensitivity of FLMAW with the FTOPSIS method. As a result, the FLMAW method provides good and robust results.

We conclude that the solutions provided by the FLMAW method are stable based on the outcomes of the rank reversal problem. The experts at FCSA will not consider the ranks produced by the FTOPSIS and FCOPRAS methods when reaching a final decision because the sensitivity analysis has demonstrated that these methods significantly lose consistency in their results.

This study is one of the important studies that addressed the subject of evaluation of digital marketing technology in I4.0 considering customer status, company status, and market information criteria by applying the LMAW methodology in fuzzy environments. However, not addressing sustainability criteria during the evaluation DMT process is one of the limitations of this research.

Future studies could enhance the digital marketing technology evaluation model in Industry 4.0 by adding more criteria in a sustainable environment, or by comparing the outcomes using alternative decision-making techniques like Fuzzy OPA, and MMD-TOPSIS models (Aouadni et al (2017), Aouadni and Euchli (2022)).

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