Journal of Ecohumanism
2024
Volume: 3, No: 8, pp. 10966 – 10980
ISSN: 2752-6798 (Print) | ISSN 2752-6801 (Online)
https://ecohumanism.co.uk/joe/ecohumanism
DOI: https://doi.org/10.62754/joe.v3i8.5705

Guided Discovery Learning and Cognitive Style: Keys to Success in Primary Geometry Education

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Abstract

This study aims to explore how the Guided Discovery Learning model and cognitive style affect geometry learning outcomes among primary school students in Cirebon City. An experimental research design with a 2 × 2 factorial structure was applied for this study. Probability sampling, specifically area cluster sampling, was used to select 167 participants. Data collection involved administering tests and questionnaires. The analysis of the data included both descriptive and inferential statistics, with prerequisite tests for normality using the Shapiro-Wilk test and homogeneity using Levene's test. Statistical significance was assessed using a two-way ANOVA, followed by post hoc Scheffé tests at a significance level of ά = 0.05. The findings revealed that: (1) students taught with the Guided Discovery Learning model had better geometry learning outcomes than those taught with the Problem-Based Learning model, (2) there was an interaction between learning models and cognitive style on geometry learning outcomes, (3) students with a field-independent cognitive style performed better when taught through the Guided Discovery Learning model compared to those using the Problem-Based Learning model, and (4) students with a field-dependent cognitive style also achieved higher geometry learning outcomes when taught using the Guided Discovery Learning model rather than the Problem-Based Learning model. Therefore, both learning models and cognitive styles significantly impact geometry learning outcomes.

Keywords: *Cognitive style, Geometry, Guided Discovery Learning, Problem-Based Learning.*

Introduction

All objects, whether naturally occurring or created by humans, have either regular or irregular shapes. These objects inherently involve aspects of mathematics, science, engineering, and art, which shape their form, structure, and design. Geometry emerges from the thought processes applied in daily life (Cherif et al., 2017). Education in geometry can enhance mathematical competence and other cognitive abilities, such as intelligence quotient (IQ), are essential factors that contribute to an individual's overall cognitive development and problem-solving skills (Clements & Sarama, 2011). Geometry, as the science of physical space, plays an essential role in children's ability to move through the world (Sinclair & Bruce, 2015).

The objectives of learning geometry include developing logical thinking, enhancing reasoning skills, improving problem-solving abilities, fostering spatial awareness, analysing, and drawing conclusions (Perry & Steck, 2015; Pack, n.d, 2016). According to (Clements & Sarama, 2011), geometry should be a focal point of education at every stage, from early childhood through to the final years of schooling, as it plays a crucial role in developing spatial reasoning and problem-solving skills. Geometry is one of the key aspects of mathematics, studied from elementary school through to higher education. The ICME study on geometry indicates that nearly every country bases its primary school students explore the study of two- and three-dimensional shapes as part of their geometry curriculum (Sinclair & Bruce, 2015). Geometry lessons are necessary, even at the elementary school level (Kuzle, 2022). In Indonesia, sixth-grade students in primary schools learn about two- and three-dimensional geometry, including concepts such as points, lines, planes, cubes, rectangular prisms, prisms, pyramids, cylinders, cones, and spheres (Alex & Mammen, 2018). Learning about 3D geometry encompasses skills like recognizing and constructing 3D shapes, drawing them, arranging 3D structures, identifying the properties of 3D geometry, and calculating the surface area and volume of three-dimensional shapes (İbili et al., 2020).

Geometry learning outcomes encompass students' cognitive, affective, and psychomotor skills developed after learning geometry. Learning outcomes serve as indicators of achieving educational goals (Ernita et al.,

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2021); (Dobbins et al., 2016). Achieving optimal geometry learning outcomes is crucial. International studies indicate a significant weakness in students' geometric performance (Sialaros, 2019). Low geometry learning outcomes can be attributed to several factors: (1) geometry instruction is often limited to verbal explanations and textbook illustrations (Clements & Sarama, 2011), (2) the teaching approach is often teacher-centred, where students are given geometry formulas directly, (3) students are less actively engaged as subjects of learning, and (4) geometry lessons tend to focus only on textbook problems and formulas. Research has shown that elementary school students often perform poorly in geometry tasks (Gracin & Kuzle, 2018; Kuzle, 2022). Observations of sixth-grade primary school students in Cirebon, Indonesia, in 2024 on the topic of 3D geometry revealed an average score of 47, which falls into the low category. Identified challenges include difficulties in recognizing parts of 3D geometry, determining 3D shape nets, drawing 3D shapes, and applying surface area and volume formulas to real-life situations. These challenges contribute to the low performance in geometry learning. One of the factors contributing to the poor geometry learning outcomes is the instructional model used. Therefore, a student-centred instructional model that demystifies the challenges of learning geometry, promotes active learning with teacher guidance, and encourages critical and independent thinking is needed. Among such models is the Guided Discovery Learning (GDL) model, which enables students to independently discover concepts while receiving guided support. Mansoor et al., 2024).

The GDL model is student-centred, with the teacher acting as a facilitator (Simamora et al., 2018). GDL is based on J. Bruner's theory, which emphasizes that the core of learning involves active transformation, where students must independently discover concepts (Lee & Yeung, 2021). This model aligns with John Dewey's theory, which posits that individuals learn best when they actively construct knowledge (Goldstein, 2016). GDL also reflects Vygotsky's theory, which suggests that learning is most effective when knowledge is collaboratively constructed among individuals with the assistance of more capable peers and teachers in problem-solving (Duke et al., 2021). GDL employs a scientific approach, where students engage in group problem-solving through structured steps including stimulating, identifying, and formulating problems, collecting data, processing information, verifying results, and drawing conclusions (Yerizon et al., 2018). This learning process is more guided, with teacher support to prepare for negative outcomes issues such as excessive cognitive load, potential misconceptions, and delivering positive reinforcement to support student growth (Simamora et al., 2018).

The advantages of using GDL in geometry learning include providing opportunities for students to make independent discoveries, promoting and enhancing spatial visualization skills, and contributing to the development of geometric understanding (Cherif et al., 2017). Research findings show that: (1) Buchbinder; (2018) demonstrated that GDL could systematically guide students in discovering circle theorems; (2) (Simamora et al., 2018) found that GDL significantly improves mathematical problem-solving abilities; and (3) Kartikaningtyas et al., (2017) concluded that GDL is more effective in enhancing geometry learning outcomes. On the other hand, a model with a relatively similar impact, the Problem-Based Learning (PBL) model, is an active learning approach where problems serve as the driving force to facilitate learning and improve geometry learning outcomes (Ayyildiz & Tarhan; 2018). Both the GDL and PBL models are external factors influencing geometry learning outcomes. Another significant factor affecting geometry learning outcomes is cognitive style.

Cognitive style refers to an individual's consistent way of thinking, acquiring, understanding, organizing, and processing information (Aggarwal & Woolleyb , 2019). It is closely related to students' learning behaviours, particularly through the concepts of Field-Dependence (FD) and Field-Independence (FI) (Y. T. Chen et al., 2019). The FD/FI cognitive styles are based on how individuals perceive their environment and provide insights into learners' social and cognitive characteristics (Ubuz & Aydınyer, 2019). Numerous studies have shown that cognitive style significantly impacts geometry learning outcomes (Nicolaou & Xistouri, 2011). Research has consistently demonstrated that cognitive styles influence learning effectiveness (S. Y. Chen & Chang, 2016). Specifically, cognitive style has been found to affect students' geometry learning results (Qolfathiriyus et al, 2019; Chrysostomou et al., 2015). Therefore, the research questions for this study are as follows:

Q1= Is there a difference in geometry learning outcomes between primary school students taught using the GDL model and those taught using the PBL model?

Q2= Is there an interaction between the learning model and cognitive style in influencing geometry learning outcomes?

Q3= Is there a difference in geometry learning outcomes between students with a Field-Independent (FI) cognitive style who learn using the GDL model and those with the same cognitive style who learn using the PBL model?

Q4= Is there a difference in geometry learning outcomes among primary school students with a Field-Dependent (FD) cognitive style when comparing those taught using the GDL model to those taught using the PBL model?

Literature Review

Cognitive Style

Riding (2014) define that cognitive style refers to an individual's preferred way of processing information, thinking, and learning, influenced by specific psychological dimensions such as visual-verbal, analytic-global, or reflective-impulsive. Cognitive styles play a crucial role in how students understand and solve problems in various learning contexts, including mathematics and science. For instance, students with an analytic style are often more adept at comprehending detailed abstract concepts, whereas those with a global style are better at grasping overarching relationships within a system (Cabrera & Cabrera, 2023). In education, recognizing cognitive styles helps teachers tailor teaching methods to support individual learning needs (Gui & Ismail, 2024). Research by Song et al. (2024) indicates that leveraging cognitive styles in teaching can enhance learning effectiveness by providing a more personalized approach. In discovery-based learning environments, reflective learners may excel in tasks requiring in-depth analysis, while impulsive learners might thrive in free exploration. By understanding students' cognitive styles, educators can design inclusive learning experiences, such as combining visual aids with textual information or offering balanced exploratory guidance (Rapanta et al., 2020).

Geometry

Geometry, a branch of mathematics, focuses on studying shapes, sizes, patterns, and spatial relationships, playing a vital role in developing students' spatial reasoning skills (Battista et al., 2018). Mastery of geometry helps students understand abstract concepts and their applications in real-life contexts, such as design, architecture, and navigation (Hwang et al., 2021). However, learning geometry can be challenging due to its reliance on complex visualizations and abstract thinking. Active learning approaches like Guided Discovery Learning can effectively aid students in understanding geometric concepts through step-by-step exploration (Gosztonyi & Varga, 2023). Studies have shown that integrating Problem-Based Learning (PBL) with geometry enhances students' problem-solving and decision-making skills (Rézio et al., 2022; Tursynkulova et al., 2023). For instance, tasks like designing a park layout or solving spatial puzzles help students gain a deeper understanding of fundamental geometric principles (Puig et al., 2022). Additionally, this approach fosters critical thinking and creativity, providing students with a more meaningful learning experience (Vincent-Lancrin, 2021).

Guided Discovery Learning

Guided Discovery Learning is a teaching approach where students are guided to uncover concepts or principles through exploration facilitated by the teacher (Janssen et al., 2014). This method blends independent learning with structured support, such as hints, questions, or examples, to help students achieve a deeper understanding. It enables learners to construct knowledge from their experiences, fostering active engagement and conceptual comprehension. In geometry, Guided Discovery Learning is often used to assist students in exploring the relationships between geometric shapes by completing exploratory tasks

(Abrahamson & Kapur, 2018). The strength of Guided Discovery Learning lies in its ability to promote critical thinking and curiosity among students (Yerimadesi et al., 2019). Research suggests that learners using this approach tend to understand and retain material better over time compared to those using direct instruction (Bakare & Orji, 2019; Tsai, 2018). However, the success of this method relies heavily on the teacher's role in providing appropriate guidance without overly restricting students' creativity, ensuring that learning objectives are met effectively.

Problem-Based Learning

According to Schwartz (2013), Problem-Based Learning (PBL) is an instructional method centered around real-world problems, where students actively engage in identifying, analyzing, and solving these challenges. This approach is designed to enhance critical thinking, problem-solving, and collaboration skills. In PBL, students typically work in small groups to develop solutions to complex issues, indirectly helping them grasp the underlying subject matter more effectively. For example, applying PBL in geometry could involve designing an efficient geometric structure for a specific project (MacLeod & van der Veen, 2020; Tursynkulova et al., 2023). PBL also promotes meaningful learning by connecting academic content to real-world applications. Studies have demonstrated that students exposed to PBL exhibit superior analytical and problem-solving abilities compared to those taught using traditional methods (Li et al., 2022; Ssemugenyi, 2023; Zhao et al., 2020). However, the success of PBL depends on meticulous planning and effective facilitation by instructors, ensuring that students not only focus on finding solutions but also thoroughly understand the relevant foundational concepts.

Material and Methods

This study uses an experimental approach with a posttest-only design, as described by Creswell (2007). This research design involves random assignment (R) for both the experimental and control groups, with the treatment (X) being applied to the experimental group. After the treatment, learning outcomes are measured through a post-test conducted on both groups. This design allows the research to test the effects of two independent variables, namely learning models and cognitive styles, on geometry learning outcomes. The factorial group design also includes two independent variables analysed together: the learning models, which consist of the GDL and the PBL Model, and cognitive styles, which consist of Field-Independent and Field-Dependent cognitive styles, with geometry learning outcomes as the dependent variable (see Table 1).

Table 1. Factorial Group Design for Research

Learning Model	A	
Cognitive Style	A_1	A_2
B_1	A_1B_1	A_2B_1
B_2	A_1B_2	A_2B_2

Description

A	: Learning Model	B	: Cognitive Style
A_1	: Guided Discovery Learning Model	B_1	: Cognitive Field-Independent Style
A2	: Problem Based Learning Model	B_2	: Cognitive Field-Dependent Style

The sampling technique employed in this study was probability sampling, specifically area cluster sampling (Sedgwick, 2013), focusing on sixth-grade primary school students. The research sample consisted of 167 students. The independent variables in this study were the learning models and cognitive styles, while the dependent variable was geometry learning outcomes. Geometry learning outcomes were measured using a test, while cognitive styles were assessed using a psychometric test. The data analysis included both descriptive and inferential statistics. The inferential analysis was conducted with the prerequisite tests for normality using the Shapiro-Wilk Test (Uthoff et al., 2020) and homogeneity using Levene's Test (Loh, 1987). Hypothesis testing was performed using a three-way ANOVA, followed by a post hoc Scheffé test with a significance level of ἀ = 0.05 (Karimi et al., 2017).

Research Instruments for Geometry Learning Outcomes

Geometry Learning Outcomes refer to changes in a student's abilities across cognitive, affective, and psychomotor domains after the process of learning geometry. The operational indicators for measuring geometry learning outcomes include: (1) remembering 3D geometry concepts. (2) understanding elements, nets of 3D geometry, and formulas for calculating volume. (3) applying formulas related to 3D geometry. (4) analyzing 3D geometric shapes. (5) evaluating 3D geometry. (6) creating equations for surface area and volume of 3D shapes.

Cognitive Style Instrument

The Cognitive Style Instrument utilized a psychometric test developed by Witkin (1977), known as the Group Embedded Figures Test (GEFT). This test comprises 25 geometric design items to assess individual cognitive perception skills, specifically the ability to identify simple shapes embedded within complex figures (Ates & Cataloglu, 2007). The GEFT consists of three sections: section 1: Contains 7 practice items to familiarize participants with the test format. These items are not included in the final GEFT score. Sections 2 and 3: Each contains 9 complex items. The scoring range for the GEFT is from 0 to 18, with each correct answer scoring 1 point and incorrect answers scoring 0. The categorization is as follows: Field-Dependent (FD): Scores between 0 and 11. And Field-Independent (FI): Scores between 12 and 18.

Results and Findings

The data analysis was conducted in three stages as follows: (1) Descriptive Statistics, (2) Inferential Statistics, which involved two testing processes: (a) the Normality Test using the Shapiro-Wilk Test. The decision criteria for determining whether a data distribution is normal are based on comparing the Asymp Sig (2-tailed) value with $\alpha = 0.05$. If Sig > α (0.05), the data is considered normal. If Sig < α (0.05), the null hypothesis (Ho) is rejected. (b) the Homogeneity Test using Levene's Test. The decision criteria state that if the significance value (Sig) is greater than $\alpha = 0.05$, the variances are considered homogeneous. The results of the Descriptive Analysis are presented in Table 2.

Table 2. Descriptive Statistics Summary of Geometry Learning Outcomes Data

Group	N	Max	Min	Range	Mean	Median	Mode	Var	Std Dev
A1	85	95	63	32	78.94	80	80	50.65	7.12
A2	82	88	58	30	72.51	73	73	31.73	5.63
B1	87	95	63	32	78.32	78	75	41.71	6.46
B2	80	90	58	32	73.08	73	73	48.33	6.95
A1B1	16	95	80	15	86.81	87	83	16.96	4.12
A1B2	16	75	63	12	68	68	63	15.8	3.97
A2B1	16	88	70	18	78	78	75	23.32	4.83
A2B2	16	75	58	17	67	67	65	22.3	4.72

Subsequently, the researcher conducted prerequisite tests using SPSS software.

Prerequisite Test for Analysis of Variance

Data Normality Test

The data normality test was performed using the Shapiro-Wilk test with a significance level of $\alpha = 0.05$. The testing criteria are Accept H_0 if Sig. > 0.05 (indicating that the data is normally distributed). Reject H_0 if Sig. < 0.05 (indicating that the data is not normally distributed). The results of the normality test are presented in the table below.

Table 3. Summary of Normality Test Results

Group	N	Shapiro-Wilk Sig	Sig	Result	Group	N	Shapiro-Wilk Sig	Sig	Result
A1	85	.068	0.05	Normal	A1B1	16	.492	0.05	Normal
A2	82	.190	0.05	Normal	A1B2	16	.174	0.05	Normal
B1	87	.321	0.05	Normal	A2B1	16	.911	0.05	Normal
B2	80	.100	0.05	Normal	A2B2	16	.841	0.05	Normal

Based on Table 3, all groups have Shapiro-Wilk test values greater than $\alpha = 0.05$; therefore, H_1 is rejected, and H_0 is accepted. This indicates that the samples come from a normally distributed population. Subsequently, the researcher conducted a homogeneity test.

Data Homogeneity Test

The homogeneity test was performed on data sets that followed a normal distribution to evaluate whether the variances across the treatment groups were equal. Levene's Test was applied for this purpose. The criteria for determining homogeneity are as follows: if the significance value is greater than 0.05, the data is considered homogeneous; if the significance value is less than 0.05, the data is deemed not homogeneous. The outcomes of this homogeneity test are shown in the table 4.

Table 4. Test of Homogeneity of Variances

Group	**Levene Statistic**	**df1**	**df2**	**Sig.**	**Descriptions**
A1 and A2	1.190	1	89	.278	Homogeny
B1 and B2	.102	1	89	.661	Homogeny
A1B1, A1B 2, A2B 1, A2B2	.478	3	60	.699	Homogeny

Table 4 shows that all learning groups obtained Sig values greater than 0.05, indicating that the data distribution is homogeneous overall. Next, the researcher tested the research hypothesis using a two-way ANOVA (parametric statistical technique) to investigate two effects: the main effect and the interaction effect. Subsequently, a Post Hoc test (Scheffe test) was conducted. The decision rule is as if Sig. $< \alpha$ (0.05), then H_0 is rejected and H_1 is accepted. If Sig. $> \alpha$ (0.05), then H_0 is accepted and H_1 is rejected. The results of the ANOVA test are shown in the following table:

Table 5. Tests of Between-Subjects Effects

Source	**Type III Sum of Squares**	**df**	**Mean Square**	**F**	**Sig.**
Corrected Model	4465.422[a]	3	1488.474	81.074	.000
Intercept	359850.016	1	359850.016	19600.341	.000
Model	1345.508	1	1345.508	71.651	.000
GK	4465.422	3	1488.474	81.074	.000
Model * GK	103.320	1	103.320	5.502	.021
Error	1101.563	60	18.359		
Total	365417.000	64			
Corrected Total	5566.984	63			

a. R Squared = .802 (Adjusted R Squared = .792)

Moreover, Table 6 provides a detailed recap of the Post Hoc Test (Scheffé Test) multiple comparisons, summarizing the significant differences found between the various groups in the study. The table highlights the pairwise comparisons made between different learning models and cognitive styles, offering insights into the specific relationships between these variables and their impact on geometry learning outcomes. Each comparison is accompanied by the corresponding statistical values, such as the mean difference, significance level, and confidence intervals, allowing for a comprehensive understanding of the results. This analysis helps to identify where the most notable differences lie, providing a deeper interpretation of the data and guiding further exploration into the factors influencing student performance.

Table 6 Recap of Post Hoc Test (Scheffe Test) Multiple Comparisons

(I) Post Hoc	(J) Post Hoc	Mean Difference	Std. Error	Sig	95% Confidence Interval		Testing	Result
					Lower Bound	Upper Bound		
A1B1	A2B1	8.44*	1.515	.000	4.08	12.80	Sig < 0,05	Significant
A1B2	A2B2	4.88*	1.515	.022	.52	9.23	Sig < 0,05	Significant

Based on observed means. The error term is Mean Square (Error) = 29.084.* The mean difference is significant at the 0.05 level.

Discussions

The Difference in Geometry Learning Outcomes between Students Learning with the GDL Model and Students Learning with the PBL Model

The two-way ANOVA test results (Table 4) reveal that the learning model significantly impacts geometry learning outcomes, as indicated by F_calculated = 71.651, F_table = 2.76, and a significance value of 0.00 (Sig. < 0.05). This shows that students who learned using the Guided Discovery Learning (GDL) model outperformed those who used the Problem-Based Learning (PBL) model. The GDL model is an instructional method that involves guiding students to independently discover solutions to problems. This model includes a sequence of steps: giving a stimulus, identifying problems, collecting and processing data, verifying, and drawing conclusions. For instance, in a geometry class, a teacher might use a physical teaching aid, like an instant noodles box as a model of a rectangular prism, to help students visualize its properties—sides, angles, and faces. This approach aligns with Piaget's theory, which emphasizes the importance of concrete objects for learning in children aged 6-12.

During the identification stage, students interact with the model to explore the net and the formulas for surface area and volume. This hands-on activity promotes better understanding and retention as students differentiate between faces, edges, and vertices, often facilitated through group discussions. Research supports that the GDL model enhances critical thinking and active participation, motivating students and fostering problem-solving skills (Sucipta et al., 2018; Shieh & Yu, 2016).

In the data collection stage, students search for information from various sources to solve given problems, which helps them deepen their understanding of concepts like rectangular prisms. As they progress to processing data, they discuss, answer questions, and engage in teacher-guided sessions that develop their analytical and evaluative skills. This stage embodies Dewey's “Learning by Doing” theory, emphasizing active student engagement to facilitate learning. Verification involves reviewing findings, such as calculating the surface area and volume of the prism and presenting group work. This reflection enables students to consolidate their understanding and encourages knowledge construction through social interaction, aligning with Vygotsky's theory on collaborative learning. The generalization phase allows students to summarize and apply what they've learned, aiding long-term retention.

Research findings affirm that the GDL model encourages students to be independent, engaged, and curious. This model helps students enjoy learning, as they experience the process of discovering concepts themselves. Studies by Siregar et al. (2019) and Winangun et al. (2021) further highlight how GDL nurtures independence, interactivity, and self-initiated inquiry, making learning more meaningful and effective. Conversely, the PBL model focuses on using problems as a stimulus for learning. The steps include presenting a problem, identifying it, gathering information, choosing a solution, and evaluating the results. However, in the PBL model, students often struggle with understanding and recalling geometric concepts, as the method lacks the use of concrete teaching aids. During group work, students may become passive due to insufficient prior knowledge and minimal teacher guidance. This can lead to conceptual errors and disorganized learning processes, as noted by Ayyildiz and Tarhan (2018) and Chan (2012).

In the PBL model, the abstract nature of learning materials and a lack of structured teacher involvement can impede students' ability to effectively analyze and evaluate geometry problems. Consequently, students may fail to present accurate solutions during group presentations, showcasing difficulties in fully grasping the material. The GDL model's advantage over the PBL model lies in its structured guidance and use of tangible aids that cater to students' developmental stages. According to Piaget, children in the concrete operational stage learn best with real objects they can manipulate, a critical component present in the GDL but less emphasized in the PBL model.

Empirical data supports these findings: students using the GDL model had higher average scores, with the highest score at 95 and a mean of 78.94, compared to PBL students, who scored a maximum of 88 with a mean of 72.51. These results align with studies such as Fitriyah (2023), demonstrating that the GDL model facilitates higher achievement in geometry compared to the PBL model. In summary, the GDL model proves more effective than the PBL model for teaching geometry. It provides a guided yet exploratory approach that allows students to actively construct knowledge, apply critical thinking, and enjoy learning. The use of concrete teaching aids, combined with teacher support and collaborative learning, leads to better outcomes and deeper understanding of geometric concepts.

The Interaction between Learning Models and Cognitive Styles on Geometry Learning Outcome

The two-way ANOVA test results indicate an interaction between learning models and Self-Regulated Learning (SRL) on geometry learning outcomes. In the AB group, F_calculated was 4.414, surpassing the F_table value of 2.76, and the significance value of 0.38 was below $\alpha = 0.05$, leading to the rejection of H0. This confirmed that both the learning model and SRL influence geometry learning outcomes. The GDL model engages students in active learning, encouraging them to build knowledge independently with limited guidance from teachers and peers. Conversely, the PBL model uses real-world problems to drive learning, fostering critical thinking and problem-solving (Dahl, 2018). However, these models may yield varying results depending on students' cognitive styles, which affect how they process, organize, and utilize information.

Cognitive styles are categorized as Field-Independent (FI) and Field-Dependent (FD). FI students are typically more autonomous, motivated by intrinsic goals, and able to focus and learn independently, while FD students are more socially oriented, rely on external motivation, and need more guidance (Aggarwal & Woolleyb, 2019; Chen et al., 2019). These traits influence how effectively they learn with different teaching models. Students with an FI cognitive style showed better learning outcomes with the GDL model than with the PBL model. This result aligns with studies such as Yerizon et al. (2018), which found that FI students excelled in environments that encouraged independent concept discovery. The GDL model allows FI students to analyze and handle abstract information effectively with teacher guidance. Conversely, PBL posed challenges for FI students due to limited geometry knowledge and less intensive teacher support, resulting in suboptimal learning outcomes.

Students with an FI cognitive style learning geometry through the GDL model were more engaged, meticulous, and less reliant on teacher intervention, facilitating better outcomes. These findings are consistent with Shieh & Yu (2016), who observed increased active participation with the GDL model. On the other hand, FD students in the GDL model needed more teacher guidance and struggled without it, leading to lower performance. This indicates that while GDL benefits FI students due to their natural inclination toward independent learning, FD students may find it challenging without sufficient support. For the PBL model, FI students also performed better than FD students. The FI cognitive style's strengths—critical, analytical, and logical thinking—enabled these students to navigate problem-solving more effectively. However, FD students faced challenges such as passive learning attitudes and higher dependence on teacher support, making it difficult for them to achieve optimal outcomes. This aligns with Chen et al. (2019), who suggested that FD students benefit more from structured, teacher-led guidance and struggle with self-directed tasks. Thus, although both cognitive styles can learn geometry with the GDL model, it is more effective for FI students who thrive on analytical tasks.

Research indicated that FD students had better outcomes with the GDL model compared to PBL, but not as high as their FI peers. The GDL approach promoted active learning and exploration, whereas the PBL method required students to identify problems and solutions independently, often without enough prior geometry knowledge or guidance, which hindered FD students' progress. According to Chan (2012), students learning with PBL can face difficulties comprehending instructions and collaborating effectively, especially when less guidance is provided.

Data from Table 1 revealed clear differences: FI students in the GDL model achieved a mean score of 87, outperforming those in the PBL model, who scored 78. FD students in the GDL model had a mean score of 68, slightly higher than those in the PBL model, who scored 66. These results illustrate the varying effectiveness of learning models based on cognitive styles. In conclusion, significant interaction exists between the learning model and cognitive style in relation to geometry learning outcomes. The GDL model supports FI students' self-directed learning, while PBL challenges them during problem identification without adequate knowledge. FD students benefit more when teacher guidance is strong, highlighting the importance of tailored instructional approaches. The two-way ANOVA's rejection of H0 emphasizes the difference in mean outcomes, but further analysis, such as a Scheffe Post Hoc Test, is needed to pinpoint specific differences between groups.

The difference in geometry learning outcomes between students with a FI cognitive style who learned with the GDL model and students with a FI cognitive style who learned with the PBL model

The seventh hypothesis test in Table 4 and the subsequent Post Hoc (Scheffe) test results for the groups A1C1 (students with an FI cognitive style learning with the GDL model) and A2C1 (students with an FI cognitive style learning with the Problem-Based Learning (PBL) model) demonstrate a significance value (Sig) of 0.00. This value is below the threshold of α (0.05), leading to the rejection of the null hypothesis (H0) and the acceptance of the alternative hypothesis (H1). This indicates that students with a Field Independent cognitive style who were taught geometry using the GDL model achieved higher learning outcomes compared to those taught using the PBL model. Therefore, it can be inferred that both the teaching model and cognitive style significantly influence geometry learning outcomes.

The GDL model emphasizes an active learning approach where students independently build knowledge through investigation, guided minimally by teachers and peers. In contrast, the PBL model uses real-world problems as the starting point for learning, encouraging students to identify, analyze, and seek solutions collaboratively. Cognitive style, which reflects how individuals think, process, and organize information, also plays a crucial role in academic outcomes. Cognitive styles are categorized into FI and FD types. FI learners tend to analyze problems carefully, enjoy structured learning environments, and are more adept at independent, analytical thinking (Ubuz & Aydınyer, 2019; Aggarwal & Woolleyb, 2019).

Students with an FI cognitive style who learned through the GDL model benefited from hands-on learning experiences during the stimulant phase. For example, they engaged in activities such as examining a model of a rectangular prism constructed from an Indomie box brought by the teacher. These students then explored the model through group discussions to identify shapes, faces, edges, and angles, enhancing their understanding of geometric concepts like cubes and rectangular prisms. This aligns with findings by Chrysostomou et al. (2015), who reported that FI learners approach problems more analytically than their FD counterparts. Furthermore, students with an FI cognitive style who used the GDL model completed tasks on their worksheets meticulously, followed teacher instructions, and compared geometry concepts, thus reinforcing their analytical thinking.

These results align with Van Hiele's geometric learning theory, which suggests that student progress through five levels of geometric understanding: holistic, analytical, abstract, deductive, and formal. Students at the initial level recognize geometric shapes as wholes without considering their properties. As they advance, they analyze properties, use logical connections, and eventually work within formal mathematical proofs (Decano, 2017). The GDL model facilitates this progression by allowing FI students to explore, observe, and gradually develop deeper geometric reasoning. The process also supports Bruner's theory of learning, which encompasses three stages: enactive (hands-on interaction), iconic (recognizing shapes and structures visually), and symbolic (using abstract symbols and formulas). For example, FI students engaged in manipulating geometric models physically (enactive), understanding images of prisms (iconic), and eventually using formulas like S^3 for volume calculation (symbolic) (Matsumoto, 2017; Lee & Yeung, 2021).

Additionally, the GDL model's emphasis on limited, guided group discussions aligns with Vygotsky's theory, which underscores the importance of social interactions in learning. Vygotsky posited that students construct knowledge more effectively through social collaboration (Duke et al., 2021). Activities in the GDL model, such as designing cube nets and constructing frames, align with John Dewey's "Learning by Doing" principle, where learners build knowledge through active participation (Goldstein, 2016). Conversely, FI students learning with the PBL model faced challenges. The open-ended nature of PBL, which encourages students to solve real-world problems through collaborative investigation, can be less effective without sufficient prior knowledge. This was evident when students struggled with unproductive group discussions, difficulties understanding task instructions, and making errors in formulating problem-solving strategies. Research supports these observations: Ayyildiz & Tarhan (2018) noted that PBL is less effective when students lack the necessary foundational knowledge, while Chan (2012) found that students may struggle with both content and collaboration in PBL settings.

Even so, students with an FI cognitive style learning with the PBL model showed some positive traits, such as being independent and focused on processing information. However, they often required more structured guidance to achieve optimal results. The preference for teacher-guided instruction in the GDL model matches FI students' natural learning style, which is characterized by independence, organization, and a focus on analytical tasks (Saracho & Witkin; Ubuz & Aydınyer, 2019). Performance differences were reflected in scores: students with an FI cognitive style using the GDL model had a mean score of 86.81, compared to 78 for those using the PBL model. This finding supports past studies, such as Fitriyah (2023), who reported similar outcomes for geometry learning, and Kartikaningtyas et al. (2017), whose research highlighted the higher effectiveness of the GDL model over PBL.

In summary, geometry learning outcomes for students with an FI cognitive style were better when they used the GDL model than when they used the PBL model. The GDL model's structured, guided approach complemented the independent, analytical nature of FI students, fostering deeper understanding and higher achievement in geometry. Meanwhile, the PBL model, while promoting collaboration and problem-solving, did not align as well with the needs of FI students, leading to less effective learning outcomes.

The difference in Geometry learning outcomes between students with an FD cognitive style who learned using the GDL model and students with an FD cognitive style who learned using the PBL model

The eighth hypothesis test results demonstrate that for student groups A1C2 and A2C2, the Post Hoc (Scheffe) test produced a significance value of 0.046, which is below α (0.05). This leads to the rejection of H0 and acceptance of H1, indicating that Geometry learning outcomes for students with a Field-Dependent (FD) cognitive style are higher when using the GDL model compared to the PBL model. This finding suggests that both learning models and cognitive styles significantly influence Geometry learning outcomes.

The GDL model encourages active student engagement by facilitating the construction of knowledge through limited guidance. In contrast, the PBL model leverages real-world problems to drive the learning process, emphasizing student analysis and problem-solving with minimal teacher support. Cognitive style, can be understood as the habitual method a person uses to analyse and organize data and using information, plays a crucial role in learning outcomes. FD students, who often rely on external goals, teacher guidance, and group work, are typically less detail-oriented and have difficulty with independent spatial analysis. Students with an FD cognitive style tend to be passive learners. However, GDL transforms their learning experience by providing structured guidance during the stimulation phase, which prompts active participation, such as observing teaching aids and asking questions. This aligns with Piaget's theory that children aged 6-12 learn effectively through concrete objects. The research shows that FD students learning Geometry through GDL actively engaged in tasks like constructing geometric shapes, supporting Dewey's "Learning by Doing" theory, which highlights the importance of active participation for deeper learning.

Bruner's learning theory further corroborates these findings by emphasizing the transformation of knowledge through enactive, iconic, and symbolic stages. The enactive stage involves direct experiences, such as using teaching aids, while the iconic stage includes visual representations like diagrams. The symbolic stage solidifies learning with abstract symbols, such as formulas. In GDL, student progress through these stages, enhancing their understanding and learning outcomes. FD students often need consistent teacher support due to difficulties completing tasks, focusing, and synthesizing information. The GDL model's structured approach, with its stages of problem statement, data collection, processing, and verification, allows teachers to provide targeted guidance. This support reduces difficulties, helping student complete tasks effectively and achieve higher learning outcomes. The GDL model also fosters collaborative discussions, aligning with Vygotsky's theory that social interaction plays a vital role in learning. Group activities help FD students share and construct knowledge, making learning more effective.

In contrast, the PBL model emphasizes independent problem-solving with minimal teacher intervention, which can be challenging for FD students. These students often struggle with critical thinking and analysis due to their reliance on external guidance and lower motivation. As a result, learning outcomes in Geometry with the PBL model are suboptimal. Previous research supports these findings, indicating that FD students are more passive, require external support, and have lower motivation, making it difficult for them to succeed in the PBL model. The study's data shows that the mean Geometry score for FD students using the GDL model was 68, compared to 66 for those using PBL. This confirms that the GDL model leads to better learning outcomes for FD students. Previous research has also supported these results, such as Fitriyah (2023), which concluded that GDL is superior to PBL for Geometry learning, and Kartikaningtyas et al. (2017), which reported mean scores of 79.13 for GDL and 72.64 for PBL. In conclusion, the findings indicate that FD students achieve better Geometry learning outcomes with the GDL model than with the PBL model. The structured guidance, active engagement, and collaborative elements of GDL cater to the needs of FD learners, enhancing their ability to process and apply information effectively. This highlights the importance of selecting appropriate teaching models that align with students' cognitive styles to maximize learning outcomes.

Conclusions

The findings of this study underscore the significant differences in geometry learning outcomes between students utilizing the GDL model and those employing the PBL model. The two-way ANOVA results demonstrated that students learning through the GDL model achieved higher scores, highlighting its efficacy in improving critical thinking and conceptual comprehension. The structured yet exploratory nature of the GDL approach, involving direct interaction with teaching aids and guided group discussions, was instrumental in helping students engage more deeply and retain geometric concepts. This aligns with Piaget’s and Vygotsky’s learning theories, which stress the importance of concrete experiences and social interaction for effective learning.

Moreover, the study revealed that cognitive style plays a critical role in influencing learning outcomes. Field Independent students, who naturally excel in analytical and self-directed tasks, thrived under the GDL model, benefiting from its clear guidance and hands-on activities. This group demonstrated a better grasp of geometry concepts through activities that supported active engagement and reflection, reinforcing Bruner’s and Van Hiele’s theories of progressive understanding. In contrast, FI students learning with the PBL model faced challenges due to its open-ended nature and reduced teacher guidance, which sometimes led to difficulties in maintaining focus and achieving clarity in problem-solving.

Therefore, the GDL model was found to be particularly advantageous for students with an FI cognitive style, promoting their ability to independently explore and solve problems. While the PBL model also encouraged critical thinking, it required a higher level of initial understanding and support that some students, particularly those with a Field Dependent (FD) cognitive style, found challenging. These insights highlight the importance of aligning teaching methods with students' cognitive preferences to optimize learning outcomes in geometry education.

Implications, Limitations, and Recommendations

The findings of this study have several implications for educational practice, curriculum development, and instructional design. First, the integration of teaching models such as GDL and PBL could significantly enhance student engagement and learning outcomes by catering to diverse cognitive styles. This insight can encourage educators to diversify their teaching approaches to accommodate both Field Independent and Field Dependent students. Additionally, curriculum developers are encouraged to design flexible learning modules that incorporate these instructional methods, fostering a more engaging, learner-focused educational setting that encourages critical thinking and problem-solving abilities. Moreover, policymakers can use these findings to advocate for professional development programs for teachers, ensuring they are well-equipped to implement these models effectively. In this way, the study not only offers a pedagogical framework for enhancing learning experiences but also highlights the importance of teacher training and curriculum design in optimizing educational outcomes.

Despite its contributions, the study has several limitations that should be considered. One limitation is the potential lack of diversity within the sample, as the results may not be fully generalizable to different educational contexts or populations. The findings might only apply to specific student demographics or institutions, and further studies are needed to assess their relevance in diverse settings. Additionally, the research may not have fully accounted for external factors such as classroom size, technological resources, or other contextual variables that could influence the success of GDL and PBL implementation. Another limitation is the potential variation in teacher expertise. The effectiveness of these models depends significantly on the teacher's ability to adapt and apply them, and the study may not have adequately addressed the challenges faced by less experienced educators in managing such dynamic teaching methods. Finally, the assessment of cognitive styles in students can be complex, and the tools used may not be universally applicable, limiting the ability to classify students' cognitive styles accurately across different contexts.

To build on the findings of this study, several recommendations are proposed for future practice and research. First, it is essential to invest in continuous professional development for teachers, ensuring they are equipped to implement GDL and PBL effectively. This training should focus on adapting these models to meet the needs of students with varying cognitive styles and developing strategies for managing diverse classroom dynamics. Educational institutions should also prioritize resource allocation, ensuring that teachers have access to the necessary tools and materials to support these teaching models. Additionally, running pilot programs within schools can be an effective strategy for evaluating the practicality of GDL and PBL in different educational settings. By gathering feedback from both educators and students, schools can refine their approaches before full-scale implementation. Long-term studies are also recommended to assess the sustained impact of these teaching models on student learning outcomes and academic achievement over time. Finally, fostering greater awareness of students' cognitive styles at the beginning of each academic year can enable teachers to tailor their instruction more effectively, ensuring that all students benefit from these innovative teaching methods.

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