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Students' Geometric Thinking Processes in Terms of Spatial Intelligence

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Abstract

Geometry is mandatory knowledge for mathematics education students who hope to become teachers. A prospective teacher must have maximum geometry skills to be able to teach geometry knowledge to students well. The observations on Mathematics Education students showed that the problem was that students' geometric thinking ability was still at level 2 (analysis). The purpose of this study is to identify the geometric thinking process of students from their spatial ability. The qualitative descriptive method used in the study was used to describe the geometric thinking process of students based on spatial ability. The subject of the study is a student in the Mathematics Education undergraduate program at a private university in Semarang, Indonesia. Research instruments used: spatial ability test, geometry test, and interview guidelines. The test and interview data results were a data triangulation to obtain qualitative descriptive analysis. The results of the study show that there are four stages of students' geometric thinking process based on their spatial abilities, namely: 1) Identify information and reflect knowledge, 2) visualize the results of identification in the form of images, and explore knowledge, 3) constructing thinking in planning solutions, 4) reasoning process to solve problems and prove statements. The results of this study can be used for future research in designing the right learning method for optimizing students' geometric thinking processes.

Keywords: *Geometric Thought Processes, Spatial Abilities, Problem-Solving.*

Introduction

Geometry is a branch of mathematical literacy that is important in visualizing objects in the real world (Arzarello et al., 2014; Alghadari et al., 2020; He et al., 2022; Setiawan et al., 2024). Geometry is mathematics that studies objects, points, lines, angles, shapes, spaces, and interrelated relationships (Ryan et al., 2008; Nusaibah et al., 2021). Informally, geometry has become known to children through the visual and manipulative objects around them daily (Aaron & Herbst, 2019; MdYunus et al., 2019; Blumenthal & Mehta, 2023; Awi et al., 2024). This situation can be beneficial for students who prefer to learn concrete math rather than learning with symbols (Clements & Sarama, 2011; Ncho et al., 2022). When studying mathematics, students must know and understand mathematical objects. There are five essential skills to master geometry: (1) visuals, the ability to recognize different shapes and spaces, the center of objects and interconnections (Lowrie & Logan, 2023); (2) verbal, the ability to classify geometric objects by name, visualizing them with verbal descriptions (Fastame, 2021); (3) drawing skills, namely the ability to sketch fields and determine specific points (Yao, 2021) ; (4) Logic skills, the ability to recognize differences and similarities between geometric objects, and be able to classify based on their properties (Trimurtini et al., 2022); dan (5) Applied skills, namely the ability to recognize physical models of geometric objects, build geometric models based on their physical objects and apply them in problem-solving (Astuti et al., 2018; Rong & Mononen, 2022). Geometry lessons emphasize the exploration of different representations, such as virtual manipulatives, written mathematical formulas, and verbal explanations, which help students build mathematical concepts and develop critical thinking (Silmi Juman et al., 2022). The purpose of learning geometry is for students to understand the properties and relationships between geometric elements to solve geometry problems (Trimurtini et al., 2021).

Geometry is introduced from an early age through flat plane shapes that children often encounter. Elementary school students learn geometry, from flat planes to building curved side spaces, and their problems in everyday life (Mawarsari, Waluya, et al., 2023). The results of the study show that students'

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geometry skills have not been maximized because students are still in Van Hiele's low level of thinking (Astuti et al., 2018; Adiastuty et al., 2020; Nusaibah et al., 2021; Matos & Souza, 2021). Problems in learning geometry experienced by students include knowing geometric shapes but not being able to recognize characteristics (Alghadari et al., 2020) and the properties of geometric shapes (Gal & Linchevski, 2010), difficulty in determining the distance between two points on a cube (Barut & Retnawati, 2020), difficulties in understanding the language of mathematics in geometry and relating it to the prior knowledge they have (Alejos, 2017). Another difficulty is that students apply geometry concepts to contextual problems (Abdullah & Wei, 2017), weak application and reasoning skills in the geometry domain (Alejos, 2017) and HOTS problem-solving (Silmi Juman et al., 2022). Students in universities also experience problems in studying geometry materials.

The results of a case study of students of the mathematics education study program at a private university in Semarang, Indonesia, show that at the level of van Hiele geometric thinking, students' geometric thinking ability is at level 2 (informal deduction) (Mawarsari, Waluya, et al., 2023). Students of the Elementary School Teacher Education Study Program at one of the state universities in Semarang, Indonesia, who received a score of geometry material above 61 (maximum score of 100) only reached 29.97%, namely 104 people out of a total of 347 students (Trimurtini et al., 2023). Based on this case shows that students' ability to solve geometry problems is still low. By the time students solve geometry problems, the student's geometry thinking process is ongoing (Mawarsari, Kintoko, et al., 2023). Not only that, the factors that affect the success of geometry learning, one of which is the geometric thinking process (Trimurtini et al., 2021).

The geometric thinking process is a stage to understand, analyze, and solve problems related to the shape, size, position, and properties of geometric objects (Siagian & Sinaga, 2019; Trimurtini et al., 2022; Mawarsari et al., 2024). This process involves a variety of cognitive skills, namely Visualization, deductive reasoning, and mental manipulation of geometric objects (Rejeki et al., 2021). The existence of Visualization shows that spatial ability plays a role in the geometric thinking process. This ability is the ability to understand, interpret, and manipulate visual and spatial information in a person's mind (Gómez-tone, 2019; Zich, 2023). A person who has spatial abilities is not only able to find patterns but also able to relate these patterns to find concepts (Steele, 2013). Spatial abilities include the mental manipulation of geometric objects, including being able to rotate, flip, or resize shapes in the mind without physical assistance (Acikgul et al., 2023). Spatial abilities are used to understand the relationships between objects in three-dimensional space, including understanding how these objects interact with each other and can be projected in two dimensions (Yuliardi & Rosjanuardi, 2021). Spatial skills are important to help students during the geometric thinking process in understanding concepts and solving problems of flat planes, building space, symmetry, transformation, and congruence in geometry (Sahrudin et al., 2022; Mjenda et al., 2023). This shows that the spatial ability possessed by students will affect the geometric thinking process of students in solving problems (Chen et al., 2020). The spatial abilities possessed by each student are undoubtedly different. One of the differences that is easy to see is when students visualize geometric shapes (Pujawan et al., 2020).

There are differences in spatial abilities possessed by students, so it is necessary to identify the geometric thinking process in solving geometric problems. The geometric thinking process that students have, whether the same or different, when viewed from their spatial ability. The stages of the resulting geometric thinking process also need to be identified based on the spatial ability of students. The findings regarding the geometric thinking process of students are found, and it is hoped that it can determine the suitable learning model and scaffolding to improve the geometry learning process in mathematics education students. So that further Research can be carried out to study, analyze, and develop learning methods and scaffolding to optimize students' geometric thinking processes based on their spatial abilities in geometry learning.

Methodology

Subject and Data Collection

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The data of this study was taken from students in the Bachelor of Mathematics Education Study Program at a private university in Semarang, Indonesia, who took the Geometry Course. This study involved thirty-two students who were given spatial ability tests. The results of the spatial ability test are then grouped into three categories: low, medium, and high. Category criteria are based on:

Table 1. Criteria for the Spatial Ability Category (Creswell, 2014)

Category	Criterion
Low	$S < \bar{x} - SD$
Keep	$\bar{x} - SD \le S \le \bar{x} + SD$
High	$S > \bar{x} + SD$

Symbol description:

S = Student Spatial Ability Test Score

$\bar{x}$ = Average spatial ability test of students

SD = Standard deviation of students' spatial ability test

Each category was taken as one research subject, so three students were obtained as research subjects. The selected research subject was then given a geometry test question.

Instruments

The research instruments used in this study are spatial ability tests, geometric thinking ability tests, and interview guidelines. The spatial ability test uses the international standard of spatial ability tests that have been translated into Indonesian. The geometric thinking ability test is carried out with content and empirical validation. This validation is carried out to find out if the instrument prepared can be used in Research. The spatial capability instrument used can be seen in the following figure.

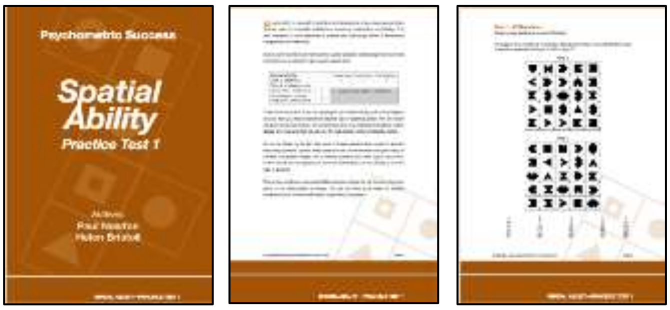

Figure 1. Spatial Ability Instruments by Paul Newton and Helen Bristoli

The above spatial capability instruments have met international standards and have been declared valid. The geometry test using instruments is arranged based on van Hiele's geometric thinking level (Salifu et al., 2018; Siagian & Sinaga, 2019; Mawarsari, Waluya, et al., 2023). The following is the geometric thinking test instrument used in the Research.

Table 2. Indicators of the Geometric Thinking Test

Question Items	Geometric Thinking Level	Geometric Thinking Level Indicator
There is a cube-shaped wood with a side length of 30 cm, which will be used as a material in making ornaments for the fence of the house.	**Level 0: Visual**	Visualize the information on the questions in the form of pictures.

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The base of the ornament is a cross-section of the cube. Let us assume that the cube is a cube that has a bottom side with corner points A, B, C, and D. The top side has corner points E, F, G, H. The cross-section of the cube is a plane that passes through points P, Q, R, and U. If point P is in the middle of the AE line, Q is in the middle of the AB line, R is in the middle of the CG line, U is in the middle of the EH line, then prove that the ratio of the volume of the cube to the volume of the space constrained by the cross-section of the cube with the vertex point F is 8 : 3! and how many wooden beams are needed to make the 12 ornaments of the fence of the house!	**Level 1: Analysis**	Identify the shape of the image based on its characteristics.
	Level 2: Formal deduction	Completing through calculation according to the concept.
	Level 3: Informal deduction	Students can prove a statement.
	Level 4: Rigour	Students can compare, analyze, and create evidence under different geometric systems.

The geometric thinking test instrument has been validated. Content validation was carried out by 4 lecturers of geometry experts. The test instrument used to find out the geometric thinking process has been validated by experts and obtained an average score of 4,675 with a Very valid category. Question validation consists of 3 aspects of assessment, namely the material aspect, the construction aspect, and the language aspect. The results of the question validation test assessment are obtained as shown in table 4 below:

Table 3. Question Validation Results

No	Question	Validator				Score Total	Average	Criteria
		I	II	III	IV			
1	S – 1	4,5	4,7	4,8	4,7	18,7	4,675	Very valid

Validation Criteria (Sukestiyarno, 2020):

1 – 1,5 = highly invalid

1,6 – 2,5 = Invalid

2,6 – 3,5 = Quite Valid

3,6 – 4,0 = Valid

4,1 – 5 = Very valid

Research Design

The research method used is qualitative descriptive (Creswell, 2012; Creswell, 2014; Sukestiyarno, 2020) by describing the results of the identification of students' geometric thinking processes reviewed from spatial abilities. The stages of Research carried out include: (1) compiling research instruments, namely grids, scoring guidelines, geometry questions, interview guidelines, and question validation sheets; (2) validating research instruments; (3) Spatial capability data capture; (4) data collection on geometric thinking processes; (5) data analysis, and (6) conclusion.

The data analysis carried out in this study follows a qualitative model procedure consisting of collecting spatial ability test data, selecting spatial ability test data, collecting geometry test data, making an analogy of geometry test data, conducting in-depth interviews about the geometric thinking process of each research subject and triangulation of data so that conclusions are obtained (Kusuma et al., 2022). The results of the geometry test were then analyzed to determine the geometric thinking process for solving the problem. The next stage is an interview to deepen the subject's thinking process in solving geometry problems. The results

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of test data, documentation, and interviews are regulated to obtain final results for drawing conclusions (Sukestiyarno, 2020).

Analysis and Results

Analysis

This research is qualitative and conducted on undergraduate students of the Mathematics Education Study Program at one of the universities in Semarang City. Based on 32 research subjects, the spatial distribution of abilities is obtained, as depicted in the figure below.

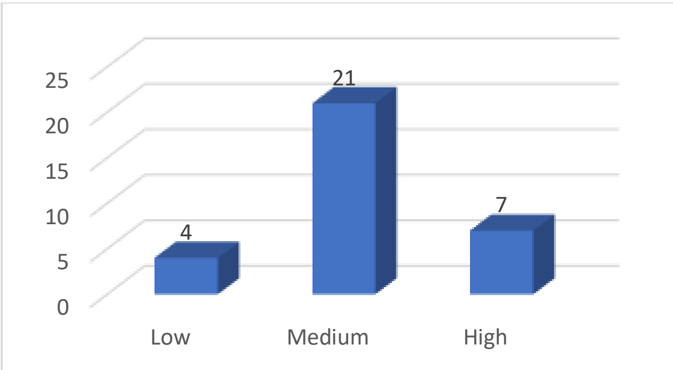
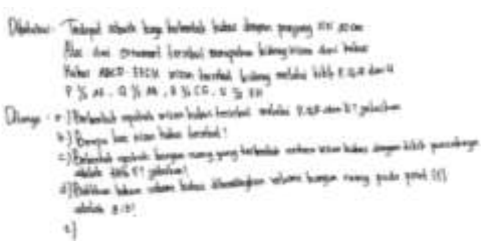


Figure 2. Grouping of Student Spatial Ability Categories

Based on Figure 2, it can be concluded that the research subjects are in the high, medium, and low categories. The average spatial ability of the research subjects was 29.6875, which was included in the medium category. Based on these categories, one subject is then selected in each category, namely 1 (one) subject in the low category with code SG1, 1 (one) subject in the medium category with code SG2, and 1 (one) subject in the high category with code SG3. The three subjects were given geometry tests of the validated instruments. The following is obtained from the geometric thinking process of each subject. The following table shows the results of the work of Geometry Subject 1 (SG1), Geometry Subject 2 (SG2), and Geometry Subject 3 (SG3).

Table 4. Geometric Thought Process Analysis 1

Research Subject	Results of Geometric Thinking	Translation
SG1		Known: A cube-shaped wood with a side length of 30 cm. An ornament with a base is a cross-section of the ABCD cube. EFGH. The cross-section passes through points P, Q, R, and U. Point P is in the middle of the AE line, Q point is in the middle of the AB line, R point is in the middle of the CG line, and U point is in the middle of the EH line. Asked: a) the cross-sectional shape of the cube through points P, Q, R, and U? b) What is the cross-sectional area of the cube? c) What is the shape of the space formed between the cube and its vertex point is point F?

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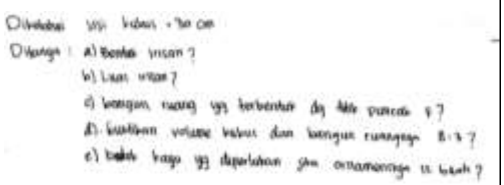
		d) Prove that the volume of the cube compared to the volume of the space at point (c) is 8 : 3!
SG2	Not writing down known and questioned statements	
SG3		Known: The length of the ribs of the cube = 30 cm Asked: a) Cross-sectional shape? b) Cross-sectional area? c) Build a space formed with its base as the cross-section and the apex point is point F. d) Prove that the ratio of the volume of the cube to the volume of the space is 8 : 3! e) The wooden blocks needed if the ornament needed were 12 pieces!

Table 5 shows that the subject carries out the initial process of solving the problem by identifying the information that is known and asked in the question. Subjects SG1 and SG3 wrote down their identification results, but SG2 did not write down their identification results. The results from the interview above showed that when SG1 read the questions, he started to think about how to solve the questions given. SG1 identifies information regarding what is known and asked in the question. Next, he recalled the knowledge he had previously learned. The solution steps considered are visualizing the problem information in the form of an image, looking for intersections, calculating the intersection area, finding the volume, and comparing two volumes. From an interview with SG2, it was found that the first step in solving the problem was reading the question and identifying the information that was known and what was asked about in the question. At this stage, SG2 has also started considering a resolution plan. SG2 assumes that he already understands and knows the information in the question so that he does not need to write known statements and ask questions to utilize the time. The interview with SG 3 was obtained by reading the questions and starting to analyze the information that was known and asked about the questions. SG3 wrote down the identification results to remember the information in the questions. Based on the information known and asked, it is also used to think about a resolution plan. The solution plan considered by both SG2 and SG3 is the same, namely: (1) draw the cube first and then place the known points so that we can find intersections through the known points using the affinity axis method, (2) determine the shape of the intersection, (3) determine the area of the sliced plane, (4) draw the pyramid referred to in the problem, (5) determine the volume of the cube, (6) determine the volume of the pyramid, (7) compare, (8) calculate the need for wooden blocks to make ornaments.

Based on test and interview document data, it was found that the three research subjects did the same thing. Namely, when the subjects read the questions, they began to understand the relevant information and recall the knowledge they had learned to solve the questions. This shows that when students read a problem, their intuition will emerge and start thinking about the best solution. At that time, they will also reflect on related knowledge. Memories of knowledge that have been learned will appear and are selected accordingly. When reading questions, identifying known and asked statements also takes place. Identification of known and asked statements can be written down or not. Some students do not write known and asked statements. However, in the student's mind, the statement remains known and asked. The identification process is important to make it easier for students to remember the essence of the question. The results of identifying known and asked statements will be used in planning a general solution to the problem. The solution plans for the three research subjects were not written down, but they had planned solutions according to their respective abilities. So, the initial process in geometric thinking is the subject's initial understanding, analysis of information, reflection on mastery of concepts or knowledge, and planning solutions in general. This process can be called information identification and knowledge reflection.

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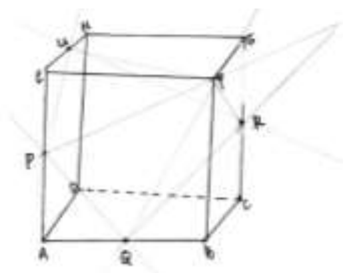
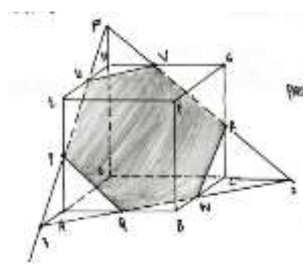
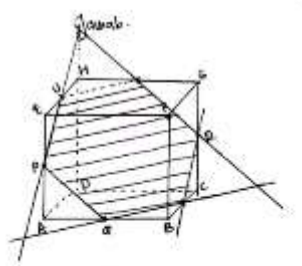
The following process is the geometric thinking process, stage 2. In the test, each research subject carries out an initial solution by describing the object according to the information obtained at the initial identification stage. Following are the results of the test documentation for each research subject

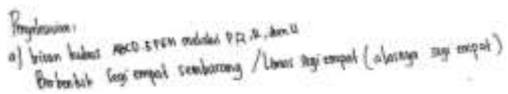
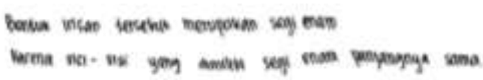
Table 5. Geometric Thought Process Analysis 2

Research Subject		
SG1	**SG2**	**SG3**
		

Based on Table 5, it shows that SG1 describes the results of the initial identification in the question. However, SG1 cannot solve the cross-sectional image of the cube through points P, Q, R, and U. Unlike SG2 and SG3, which can visualize the initial identification process and proceed with the problem-solving in describing the cross-section of the ABCD cube. EFGH through P, Q, R, and U. The results of the interview showed that SG1 could visualize the ABCD cube. EFGH and place the points P, Q, R, and U on the cube. However, it cannot describe the cross-section of the cube that passes through points P, Q, R, and U. This is because the subject SG1 cannot remember the concepts that have been learned. During the learning process regarding the concept of spatial cross-sections, SG1 was still confused and did not understand the explanation given. Meanwhile, in SG2 and SG3, it is easy to visualize the results of the initial identification well and can describe the cross-section of the cube that passes through points P, Q, R, and U.

Based on the results of the test documentation and in-depth interviews with SG1, SG2, and SG3, it is concluded that the process of visualizing the information on the questions in the form of pictures is very important. This is to make it easier for the subject to solve problems. Visualizing the results of the initial identification also requires knowledge or concepts that have been learned before. Visualizing initial identification in the form of images can be called Visualization. The stages of the geometric thinking process carried out by research subjects can then be documented as follows.

Table 6. Geometric Thought Process Analysis 3

Research Subject	Results of Geometric Thinking	Translation
SG1		Solution: Cross-section of the ABCD cube. EFGH through P, Q, R, and U in an arbitrary rectangle
SG2		The cross-sectional shape of the cube is hexagonal because the sides of the hexagon are the same length. View APQ triangle

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		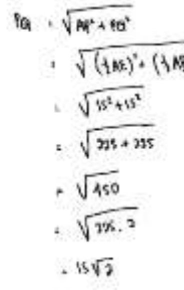
SG3		The hexagonal shape is regular, as each side is the same length.

Table 6 shows that SG1, SG2, and SG3 wrote their respective answers. The answers they wrote were based on the analysis of the drawings they had made in the previous stage. In SG1, the analysis results are written as a cross-section formed in the shape of a rectangle. This statement is false. Meanwhile, the results of the analysis of the SG2 and SG3 images stated that the cross-section formed was an equilateral hexagon.

As a result of an in-depth interview with SG1, it was found that SG1 determined the shape of the slice based on the image that had been made, namely an arbitrary rectangle. The answer is by the picture from SG1, but the statement is wrong. The mistake made by SG1 was because he did not understand how to determine the intersection of spatial shapes. SG1 experienced confusion when the lecturer presented material on geometric shapes, which had an impact when this test was given. SG1's ability to visualize information during initial identification cannot be completed properly. This is also due to the spatial abilities possessed by SG1. Even though SG1 is included in the moderate spatial ability category, SG1 has the lowest score in the medium category. Meanwhile, the results of the SG2 interview showed that the answers were also based on the results of the analysis of the images that had been made. SG2's answer regarding the shape of the geometric shape that is formed is a hexagonal plane that has the same side lengths, namely $15\sqrt{2}$ cm. The analysis carried out by SG2 was correct, using the Pythagorean theorem to determine the side lengths of the hexagon. SG2 thinks that the length of the other side is also the same length $15\sqrt{2}$ cm. So, what SG2 believes is that the intersection area is a regular hexagon. However, SG2 doesn't write it in full. The results of the interview with SG3 showed that the answer given was based on the drawing that was made, which was in the form of a regular hexagon and included the reason, namely because it had the same side lengths. The analysis carried out by SG3 is different from that of SG2. The analysis carried out by SG3 was not calculated directly, only analyzing in his mind that PQ was the hypotenuse of triangle APQ, where the length AP = ½ length AE, and length AQ = ½ length AB even though AE and AB are cube edges, namely 30 cm. So, AP = AE = 15 cm. Likewise with the other side of the hexagon.

Based on the test and interview results, it can be concluded that each research subject's analysis reveals a different understanding. The results of each analysis are based on the visualizations that have been made. The Visualization of the slice field that forms affects the way the research subject thinks and recalls the knowledge or concepts they have previously acquired. The stages of the research subject's geometric thinking process can then be documented as follows.

Table 7. Geometric Thinking Process Analysis 4

Research subject	Geometric Thinking Results	Translation

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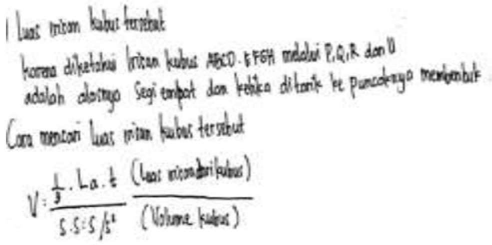
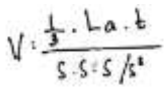
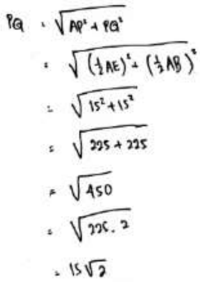
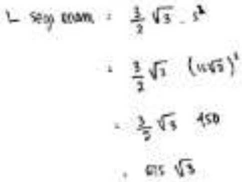
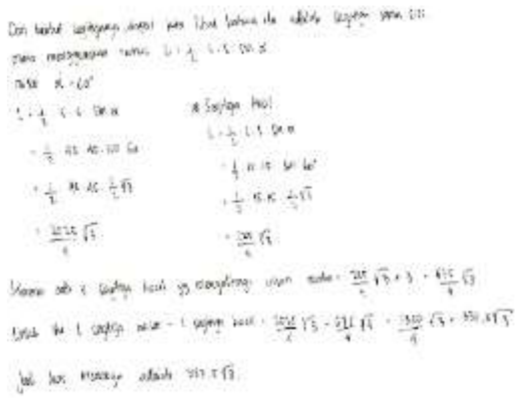
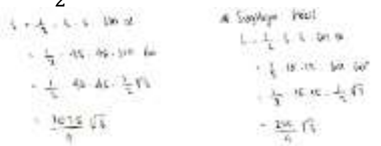
SG1		The area of the intersection of the cube ABCD.EFGH. Because it is known that the intersection of the cube is ABCD.EFGH through P, Q, R, and U is a quadrangle at the base and forms when drawn to the top. How to find the volume of the cube. 
SG2		See triangle APQ  The area of a regular hexagon is  The area of the slide cube is $675\sqrt{3}\ cm^2$
SG3		From the shape of the triangle, we can see that it is an equilateral triangle, so use the formula. $L = \frac{1}{2} s.s.\sin\alpha$ dengan $\alpha = 60^0$  Because there are three small triangles surrounding the slice, then $\frac{225}{4}\sqrt{3} \times 3 = \frac{675}{4}\sqrt{3}$ For that reason, L is a large triangle – L is a small triangle. $\frac{2025}{4}\sqrt{3} - \frac{675}{4}\sqrt{3} = \frac{1350}{4}\sqrt{3}$ $= 337{,}5\sqrt{3}$ So the area of the slice is $337{,}5\sqrt{3}$

Based on table 7 shows that SG1, SG2, and SG3 determine the area of the geometric shape using different methods. SG1 finds the area of the intersection of the geometric shape using the ratio of the volume of the cube to the volume of the pyramid, which has the base of the intersection of the geometric shape. However, in this way, SG1 cannot continue with the solution.

The analysis of tests and interviews with SG1, SG2, and SG3 showed that the three had different ways of finding the area of a geometric intersection. SG1 uses volume comparisons, but this cannot be solved

Journal of Ecohumanism
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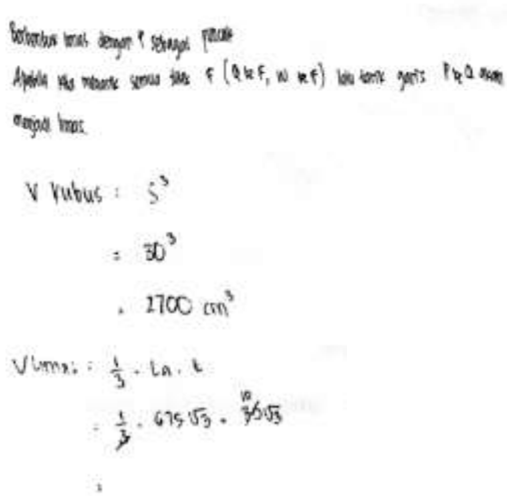
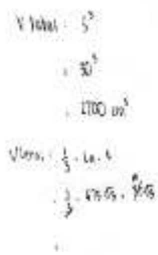
because they feel confused and cannot think about the concepts used to solve the geometric problem. SG2 uses a formula $L = \frac{3}{2}\sqrt{3}s^2$. When solving these questions, SG2 explored the knowledge they had gained while studying at the school. At the junior high school level, SG2 gained knowledge about the formula for the area of a regular hexagon-namely $L = \frac{3}{2}\sqrt{3}s^2$. This means that in solving geometric problems, previously acquired knowledge is also required. SG2 succeeded in generating long-term memories regarding the concept of the area of a hexagon. Meanwhile, SG3 uses a different method from SG1 and SG3, namely using the sine rule to find the area of a hexagon, $L = \frac{1}{2}s.s.\sin\alpha$. The solution method used by SG3 is almost the same as SG2, namely exploring the appropriate knowledge they previously had. It's just that SG3 uses trigonometry concepts. The trigonometric formula used is the area of a triangle if two sides are known, and an angle is enclosed by the two sides, so the area of the triangle used is $L = \frac{1}{2}s.s.\sin\alpha$. The area of this triangle is used because the intersection area formed is a regular hexagon consisting of 6 equilateral triangles. The area of the hexagon is six times the area of the triangle. However, in its solution, SG3 could not answer correctly due to an error in determining the length of the side of the triangle. The length of the side of the triangle written is 45 cm. This is what causes the final answer from SG3 regarding the area of the intersection of spatial shapes to be incorrect, name 337,5 $\sqrt{3}\ cm^2$. The stages of the geometric thinking process carried out by research subjects can then be documented as follows.

Table 8. Geometric Thinking Process Analysis 6

Research subject	Geometric Thinking Results	Translation
SG1	Not answered (Cannot prove that the ratio of the volume of the cube to the volume of the geometric figure at the top of point F and the base of the intersection through P, Q, R, S is 8 : 3)	Not answered (Cannot prove that the ratio of the volume of the cube to the volume of the geometric figure at the top of point F and the base of the intersection through P, Q, R, S is 8 : 3)
SG2		It is pyramid-shaped with F as the peak. It will become a pyramid if we draw all points F (Q to F, W to F) and then draw the line P to Q. 

Journal of Ecohumanism
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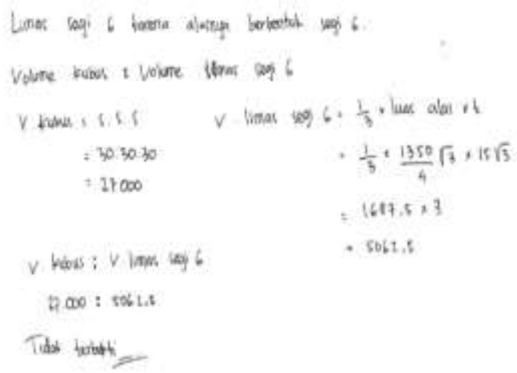
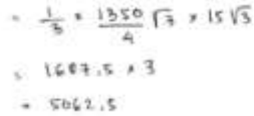
SG3		A hexagon pyramid because its base is a hexagon. Cube volume: the volume of a hexagonal is Volume of cube = s x s x s = 30 x 30 x 30 = 27,000 The volume of a hexagon pyramid = 1/2 base area x height  Cube Volume : Volume Limas = 27.000: 5062, 5 So it is not proven that the volume of the cube : volume of limas is 8 : 3.

This stage is the final stage in proving the truth of the statement. Table 8 shows that SG1 cannot prove the statement. SG2 proves the comparison of the volume of a cube and the volume of a geometric shape that passes through point F, and the base is the intersection of the geometric shape found in the previous stage (in the form of a regular hexagon). The way SG2 does this is by first finding the volume of each geometric shape. It's just that SG2 does not complete the calculations to determine the volume of the geometric shape that passes through point F, and the base is the intersection of the geometric shape (regular hexagon).

Meanwhile, SG3 uses the same method, namely finding the volume of each shape first and then comparing it. It's just that, in the previous stage, SG3 made a mistake in determining the length of the side of the hexagon, resulting in the area of the intersection area of the shape (the area of the regular hexagon) also being wrong. The analysis of tests and interviews with SG1, SG2, and SG3 showed that SG1 could not complete the proof because there were problems understanding the concepts of flat and space geometry. SG2 and SG3 have the same way of proving statements by finding each geometric figure's volume first. It's just that SG2 does not complete the calculations to determine the volume of the geometric shape that passes through point F, and the base is the intersection of the geometric shape (regular hexagon). SG2 is because there is a time limitation. SG2 needs time to construct and explore its knowledge in solving geometric problems. Meanwhile, SG3 has the same method as SG2, namely finding the volume of each shape first and then comparing it. It is just that, in the previous stage, SG3 made a mistake in determining the length of the side of the hexagon, resulting in the area of the intersection area of the shape (the area of the regular hexagon) being also wrong. The following is an identification of the student's geometric thinking process, which was reviewed based on his spatial ability resulting from the Research.

Table 9. Identify Geometric Thinking Processes Based on Spatial Abilities

Geometric Thought Process	Spatial Ability of Students		
	Low	Medium	High
Stage 1: Identify the information in the question and reflect on the knowledge	The stage of identifying information in the problem can be done well and reflect on the knowledge that has been learned, but it does not succeed in thinking about the right knowledge in solving the problem.	The stage of identifying information in the questions can be done well and successfully reflect knowledge in planning problem-solving. However, the identification results are not written down due to the efficiency of the completion time.	The stage of identifying information in the questions can be done well and successfully reflect knowledge in planning problem-solving. The identification results are written to make it easier to remember the problems in the questions.

Journal of Ecohumanism
2024
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Stage 2: visualize the results of identification in the form of images and dig up knowledge	At this stage, the subject can visualize the results of identifying the information in the problem but cannot visualize the solution to the problem because he fails in digging up knowledge to solve the problem.	This stage can be done by the subject, both visualizing the results of the identification of the problem and visualizing the solution to solve it. This is characterized by the subject being able to visualize a known image, namely the ABCD cube. EFGH and put known dots on the cube. Furthermore, it can carry out the visualization process in determining the cross-section of the cube that passes through known points.	This stage can be done by the subject, both visualizing the results of the identification of the problem and visualizing the solution to solve it. This is characterized by the subject being able to visualize a known image, namely the ABCD cube. EFGH and put known dots on the cube. Furthermore, it can carry out the visualization process in determining the cube slices that pass through known points.
Stage 3: constructing thinking in planning the completion	This subject experiences difficulties, so the construction stage in solving the problem cannot be carried out.	The research subject can construct his or her thinking in planning a solution by reflecting on the knowledge that has been acquired or learned.	The research subject can construct his or her thinking in planning a solution by reflecting on the knowledge that has been acquired or learned.
Stage 4: reasoning to solve and/or prove problems	The subject is weak in reasoning so he cannot solve the problem.	At this stage, the subject can reason logically in solving problems.	This stage is the use of knowledge in solving problems chosen by the subject by correct reasoning and logic. It's just that mistakes occur when determining the length of regular hexagonal ribs, the reasoning carried out is not appropriate.

Results

In the geometry problem given, each research subject with different spatial abilities has a different geometric thinking process. The given problem requires a geometric thinking process leading to its solution. For SG1, SG2, and SG3 students, the geometric thinking process begins by reading questions to identify known statements and questions that must be solved. At that stage, not only identify the information in the question but also start thinking about a solution plan. Furthermore, students recall the knowledge that has been learned previously, which can be used to solve the problems given. SG1 and SG3 write down the identification results that have been obtained to use them as a basis for solving problems and remembering so that there is no need to read the questions again (Purnomo et al., 2022). SG2 does not write the results of the identification of the information in the question, so to take advantage of the time, there is no need to write a statement of knowledge and questioning. In addition, there is no command to write down known and questioned statements. The identification results from the known and questioned statements will be used in planning a general solution to the problem (Liljedahl & Cai, 2021; Hourigan & Leavy, 2022; Purnomo et al., 2024). The three research subjects' solution planning was not written, but they had planned the solution according to their respective abilities.

The second geometric thinking process carried out by SG1, SG2, and SG3 transforms the results of identifying information that is known in visual form. This stage can be performed by SG1, SG2, and SG3 well. This means that all three can visualize in the form of building a space based on the initial information

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2024
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on the question. The results of the visualization process are used in designing problem-solving or proving geometry (Elmedina Nikoçeviq-Kurti, 2022). At this stage, in addition to visualizing the identification of the information known, the research subject also begins to think about visualizing the information towards the question. This process cannot be passed well by SG1 because it cannot remember the concepts that have been learned before. SG1 is a research subject that has minimal spatial ability in the medium category.Low spatial ability has an impact on difficulty mastering concepts related to images and difficulty solving problems visually (Bintoro et al., 2022). This visualization geometric thinking process is very important to carry out the next stage of completion (Diakité & Zlatanova, 2018; Fastame, 2021).

The third geometric thought process carried out by SG1, SG2 and SG3 is to construct their thoughts in proper completion. At this stage, SG1 could not do well. SG1 is unable to dig up the appropriate knowledge to solve the problem. The results of the study showed that SG1 did not understand the concept of geometry well. The effort made by SG1 is to ask the lecturer for an explanation again about the geometry concept presented. SG1 has not been able to interpret the geometry concept well given. SG1 needs to loop the material again to be able to understand the concept of geometry well (D. Ferdiani et al., 2022). This is in contrast to SG2 and SG3, which can construct their thinking and explore the knowledge they have learned as a basis for solving problems. The spatial ability possessed by SG2 is in the medium category, and SG3 has high spatial ability, making it easy for them to understand geometric materials related to visuals. Construction in preparing a solution plan is important to find solutions to problems (Santos-Trigo et al., 2008; MdYunus et al., 2019). The completion plan that was carried out included: (1) drawing a cube first and then placing the known points so that slices can be found through known point points using the affinity axis method, (2) determining the shape of the slices, (3) determining the area of the slice plane, (4) drawing the pyramid referred to in the problem, (5) determining the volume of the cube, (6) determining the volume of the pyramid, (7) comparing, (8) calculating the need for wooden blocks to make ornaments. The difference between SG2 and SG3 is in the formula used to determine the cross-sectional area formed (regular hexagon).

The fourth geometric thinking process is reasoning. At this stage, SG1 cannot reason in solving problems. There is a lack of understanding of SG1 about geometric materials. Meanwhile, SG2 and SG3 performed the reasoning stage well. This is seen from the ability of SG2 and SG3 in the problem-solving process. SG2 and SG3 can construct their thinking in solving these problems. It's just that SG2 and SG3 have different ways of determining the side length of the plane of the formed space-building slice (regular hexagon). The reasoning carried out by SG2 is precise, so the length of the hexagon side is correct according to the size. SG3's reasoning in determining the length of the sides of a regular hexagon is not correct because the assumptions made by SG3 are incorrect. The next reasoning process is to determine the area of a regular hexagon. SG2 can explore the knowledge that has been obtained about the formula for the area of the regular hexagonal field so that it applies the formula in finding the final result. Meanwhile, SG3 gets the final result from the area of the regular hexagon plane by reasoning that the regular hexagon comes from 6 equilateral triangles. So, the area of six is six times the area of an equilateral triangle. However, there was an error in reasoning when determining the length of the side of the triangle, resulting in the final result made by SG3 not being suitable.

The reasoning process certainly contributes to problem-solving. Every student can use various ways to learn to reason (Pujawan et al., 2020; R. D. Ferdiani et al., 2022). The reasoning process carried out by each individual will be different from each other (Ayuningtyas et al., 2019). The reasoning process is adjusted to the ability to receive and interpret knowledge that has been possessed before (Parr et al., 2021). If someone can understand and interpret, the existence of logical reasoning from students makes it easier to solve problems or prove (Baumanns & Rott, 2022). The stages of the geometric thinking process carried out by students are generally the same, it's just that there is a difference in spatial ability, so the thinking process carried out is not optimal (Fuys et al., 2013; Fastame, 2021; Macchitella et al., 2023). Students with low spatial skills are not optimal in carrying out geometric thinking processes. This suggests that weaknesses in spatial capabilities will have an impact on visualizing statements in image objects (Pujawan et al., 2020; Moritz & Youn, 2022). Geometry, which is always related to visual objects as the first step in completion, will have an impact on students who have low spatial ability (Uwurukundo et al., 2022; Rong & Mononen,

Journal of Ecohumanism
2024
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2022). Based on the findings in this study, it was obtained that the geometric thinking process of students in solving geometric problems, namely:

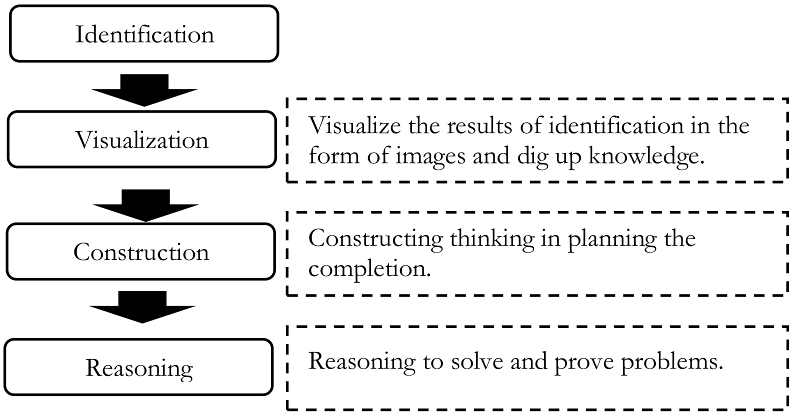

Figure 3. Geometric Thinking Process Based on Spatial Ability

Figure 3, shows that the first geometric thinking process is identification. Students carry out the identification process to find information and answer questions. This finding is in line with the problem-solving stage in Polya, which is the stage of understanding the problem (Polya, 1978; Purnomo et al., 2024b). At this stage, students have also begun to explore the knowledge they have related to the problems in the problem. The following process is Visualization. This process is the process of abstracting the results of information in the identification process in the form of visual images. These findings align with van Hiele's geometric thinking ability at the visualization level (Trimurtini et al., 2022). The third process is construction. This process is a process of constructing students' knowledge when planning a solution. This process is in line with the problem-solving stage in polya, which is the stage of designing a solution (Polya, 1978; Purnomo et al., 2024b). The last process is reasoning. In this process, students reasoned based on the construction process of the problem-solving plan. This process aims to solve or prove a given geometry problem. This process is indispensable in solving problems in van Hiele's geometric thinking ability at the level of deduction to rigor (Trimurtini et al., 2022) Based on the thinking process findings, further Research is needed to find the proper geometric thinking process for solving geometry problems properly and correctly.

Conclusion

The conclusion obtained from this study is that geometric solutions in students who have spatial abilities in each category are different. In students who have low spatial ability, they cannot solve geometry problems well. The difficulty of solving it begins with the inability to explore the knowledge learned as the basis for the solution. These difficulties have an impact when visualizing the geometry completion, which is key in the later stages of completion. Students who have moderate spatial ability but get the maximum score can solve geometry problems well, but there is a time limit, so it cannot be solved optimally. Students with moderate spatial skills begin the geometric thinking process by reading questions to identify known information and be asked. This process is done very well and is the basis for planning the next completion. The student did not write the identification results. Students consider that the most important thing is to understand the problem well and be able to construct their thinking in solving problems. In students who have high spatial ability, errors occur when reasoning in determining the length of the cross-sectional side (hexagon). This results in the problem being solved incorrectly. The study results show that students with different spatial abilities have different levels of geometric thinking. However, in general, the geometric thinking process of the three is the same. Thus, the researcher of the construction of student thinking processes is reviewed from his spatial ability consisting of 4 stages, including 1) identifying information and reflecting on knowledge, 2) visualizing the results of identification in the form of images, and exploring appropriate knowledge in planning solutions, 3) constructing thinking in planning solutions, 4) reasoning processes to explain and prove. However, the geometric thinking process obtained from the study results shows that the three research subjects cannot solve the problem completely.

The identification of the geometric thinking process reviewed from the spatial ability that has been produced can be used as a reference in developing Research to design the right model of propagation and scaffolding in geometry lectures. In addition, the identification results also show that the geometric thinking process that the research subject has carried out has not been maximized, so it is necessary to study from various other references to optimize the geometric thinking process of students in solving geometric problems.

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References

Aaron, W. R., & Herbst, P. G. (2019). The teacher's perspective on the separation between conjecturing and proving in high school geometry classrooms. Journal of Mathematics Teacher Education, 22(3), 231–256. https://doi.org/10.1007/s10857-017-9392-0

Abdullah, M. F. N. L., & Wei, L. T. (2017). Kesahan dan Kebolehpercayaan Instrumen Penilaian Kendiri Pembelajaran Geometri Tingkatan Satu (Learning Form One Geometry: Validity an Reliability of a Self-Evaluation Instrument). Malaysian Journal of Learning and Instruction, 14(Vol. 14, No. 1 June 2017), 211–265. https://doi.org/10.32890/mjli2017.14.1.9

Acikgul, K., Sad, S. N., & Altay, B. (2023). Spatial ability test for university students: Development, validity and reliability studies. International Journal of Assessment Tools in Education, 10(1), 76–97. https://doi.org/10.21449/ijate.1102435

Adiastuty, N., Agoestanto, A., & Masitoh, S. (2020). The development of the spatial visual-oriented geometry test to measure the creative thinking skills of elementary students The development of the spatial visual-oriented geometry test to measure the creative thinking skills of elementary students. Journal of Physics: Conference Series PAPER. https://doi.org/10.1088/1742-6596/1918/4/042068

Alejos, H. (2017). The Students' Difficulties in Completing Geometry Items of National Examination. International Journal on New Trends in Education and Their Implications, 8(October), 28–41. http://www.albayan.ae

Alghadari, F., Herman, T., & Prabawanto, S. (2020). Factors Affecting Senior High School Students to Solve Three-Dimensional Geometry Problems. International Electronic Journal of Mathematics Education, 15(3), em0590. https://doi.org/10.29333/iejme/8234

Arzarello, F., Bairral, M. A., & Dané, C. (2014). Moving from dragging to touchscreen: Geometrical learning with geometric dynamic software. Teaching Mathematics and Its Applications, 33(1), 39–51. https://doi.org/10.1093/teamat/hru002

Astuti, R., Suryadi, D., & Turmudi. (2018). Analysis on geometry skills of junior high school students on the concept congruence based on Van Hiele's geometric thinking level. Journal of Physics: Conference Series, 1132(1). https://doi.org/10.1088/1742-6596/1132/1/012036

Awi, Naufal, M. A., Sutamrin, & Huda, M. (2024). Enhancing Geometry Achievement in Pre-Service Mathematics Teachers: The Impact of a Scaffolded Flipped Classroom Using a Learning Management System. Journal of Ecohumanism, 3(6), 637–645. https://doi.org/10.62754/joe.v3i6.4035

Ayuningtyas, W., Mardiyana, & Pramudya, I. (2019). Analysis of student's geometry reasoning ability at senior high school. Journal of Physics: Conference Series, 1188(1). https://doi.org/10.1088/1742-6596/1188/1/012016

Barut, M. E. O., & Retnawati, H. (2020). Geometry learning in vocational high school: Investigating the students' difficulties and levels of thinking. Journal of Physics: Conference Series, 1613(1). https://doi.org/10.1088/1742-6596/1613/1/012058

Baumanns, L., & Rott, B. (2022). Identifying Metacognitive Behavior in Problem-Posing Processes: Development of a Framework and a Proof of Concept. International Journal of Science and Mathematics Education, 21(5), 1381–1406. https://doi.org/10.1007/s10763-022-10297-z

Bintoro, H. S., Sukestiyarno, Y. L., Mulyono, M., & Walid, W. (2022). The spatial thinking process of the field-dependent students in reconstructing the geometrical concept. International Journal of Evaluation and Research in Education, 11(3), 1116–1124. https://doi.org/10.11591/ijere.v11i3.22399

Blumenthal, E., & Mehta, P. (2023). Geometry of ecological coexistence and niche differentiation. BioRxiv, 4(05–02), 2023.04.21.537832. https://doi.org/10.1101/2023.04.21.537832

Chen, Y. C., Yang, F. Y., & Chang, C. C. (2020). Conceptualizing spatial abilities and their relation to science learning from a cognitive perspective. Journal of Baltic Science Education, 19(1), 50–63. https://doi.org/10.33225/jbse/20.19.50

Clements, D. H., & Sarama, J. (2011). Early childhood teacher education: The case of geometry. Journal of Mathematics Teacher Education, 14(2), 133–148. https://doi.org/10.1007/s10857-011-9173-0

Creswell, J. W. (2012). Educational Research : planning, conducting, and evaluating quantitative and qualitative research (Paul A. Smith (ed.); 4th ed.) (P. Education (ed.)). Pearson Education.

Creswell, J. W. (2014). Research Design Qualitative, Quantitative, and Mixed Method Approaches (Fourth Edi) (Paul A. Smith (ed.); 4th ed.). SAGE Publications, Inc.

Diakité, A. A., & Zlatanova, S. (2018). Spatial subdivision of complex indoor environments for 3D indoor navigation. International Journal of Geographical Information Science, 32(2), 213–235. https://doi.org/10.1080/13658816.2017.1376066

Elmedina Nikoçeviq-Kurti. (2022). The Role of Visual Representations in Geometry Learning. European Journal of Educational Research, 11(3), 1245–1257. https://www.researchgate.net/profile/Suntonrapot-Damrongpanit/publication/356662582_Effects_of_Mindset_Democratic_Parenting_Teaching_and_School_Environment_on_Global_Citizenship_of_Ninth-grade_Students/links/61a6dda685c5ea51abc0f7b6/Effects-of-Mindset-Dem

Fastame, M. C. (2021). Visuo-spatial mental imagery and geometry skills in school-aged children. School Psychology International, 42(3), 324–337. https://doi.org/10.1177/0143034321992458

Ferdiani, D., Rosita, Manuharawati, & Khabibah, S. (2022). Activist learners' creative thinking processes in posing and solving geometry problem. European Journal of Educational Research, 11(1), 117–126. https://doi.org/10.12973/eu-jer.11.1.117

Ferdiani, R. D., Manuharawati, & Khabibah, S. (2022). Activist Learners' Creative Thinking Processes in Posing and Solving Geometry Problem. European Journal of Educational Research, 11(1), 1–16. https://www.researchgate.net/profile/Suntonrapot-Damrongpanit/publication/356662582_Effects_of_Mindset_Democratic_Parenting_Teaching_and_School_Environment_on_Global_Citizenship_of_Ninth-grade_Students/links/61a6dda685c5ea51abc0f7b6/Effects-of-Mindset-Dem

Fuys, D., Geddes, D., & Tischler, R. (2013). The Van Hiele Model of Thinking in Geometry Among Adolrscents. Journal for Research in Mathematics Education, 3(1988). https://www.jstor.org/stable/749957

Gal, H., & Linchevski, L. (2010). To see or not to see: Analyzing difficulties in geometry from the perspective of visual perception. Educational Studies in Mathematics, 74(2), 163–183. https://doi.org/10.1007/s10649-010-9232-y

Gómez-tone, H. C. (2019). Impacto de la Enseñanza de la Geometría Descriptiva usando Archivos 3D-PDF como Entrenamiento de la Habilidad Espacial de Estudiantes de Ingeniería Civil en el Perú Impact of Descriptive Geometry ' s Teaching using 3D -PDF Files as Spatial Skill Training . Formación Universitaria, 12(1), 73–82.

He, Y., Yu, H., Cai, H., Peng, Z., Liu, W., & Hu, Z. (2022). Shape computing: A new geometric computing mechanism. Journal for Geometry and Graphics, 26(1), 101–114.

Hourigan, M., & Leavy, A. M. (2022). Elementary teachers' experience of engaging with Teaching Through Problem Solving using Lesson Study. Mathematics Education Research Journal, 0123456789. https://doi.org/10.1007/s13394-022-00418-w

Kusuma, D., Sukestiyarno, Y. L., Wardono, & Cahyono, A. N. (2022). The characteristics of mathematical literacy based on Students' executive function. European Journal of Educational Research, 11(1), 193–206. https://doi.org/10.12973/eu-jer.11.1.193

Liljedahl, P., & Cai, J. (2021). Empirical research on problem solving and problem posing: a look at the state of the art. ZDM - Mathematics Education, 53(4), 723–735. https://doi.org/10.1007/s11858-021-01291-w

Lowrie, T., & Logan, T. (2023). Spatial Visualization Supports Students' Math: Mechanisms for Spatial Transfer. Journal of Intelligence, 11(6), 127. https://doi.org/10.3390/jintelligence11060127

Macchitella, L., Tosi, G., Romano, D. L., Iaia, M., Vizzi, F., Mammarella, I. C., & Angelelli, P. (2023). Visuo-Spatial Working Memory and Mathematical Skills in Children: A Network Analysis Study. Behavioral Sciences, 13(4). https://doi.org/10.3390/bs13040294

Matos, A. P. P., & Souza, V. S. (2021). Teaching spatial geometry with geogebra : can it improve the problem-solving skills of prospective mathematics teachers ? Teaching spatial geometry with geogebra : can it improve the problem-solving skills of prospective mathematics teachers ? Journal of Physics: Conference Series, 8–14. https://doi.org/10.1088/1742-6596/1918/4/042082

Mawarsari, V. D., Kintoko, Zaenuri, Kharisudin, I., & Aziz, A. (2023). Students' Errors in Solving Geometry Problems of Van Hiele Levels Based on Newman's Error Hierarchy Model. Proceedings of the 2nd UPY International Conference on Education and Social Science (UPINCESS 2023), 0(Upincess), 316–323. https://doi.org/10.2991/978-2-38476-176-0_45

Mawarsari, V. D., Rochmad, Purnomo, E. A., & Arum, P. R. (2024). Profile of Van Hiele Geometry Thinking Based on Student. AIP Conference Proceedings, 020016.

Mawarsari, V. D., Waluya, S. B., & Dewi, N. R. (2023). Profile of Students ' Geometric Thinking Ability in Terms of Van Hiele Level. Proceeding of 1st Lawang Sewu International Symposium on Humanities and Social Sciences 2022 (Lewis 20200), 1, 109–117. https://doi.org/10.2991/978-2-38476-078-7

MdYunus, A. S., Ayub, A. F. M., & Hock, T. T. (2019). Geometric thinking of Malaysian elementary school students. International Journal of Instruction, 12(1), 1095–1112. https://doi.org/10.29333/iji.2019.12170a

Mjenda, M., Mutarutinya, V., & Owiti, D. (2023). Assessing the Effectiveness of Computer-Aided Instructional Techniques in Enhancing Students' 3D Geometry Spatial Visualization Skills Among Secondary School Students in Tanzania. International Journal of Learning, Teaching and Educational Research, 22(6), 613–637. https://doi.org/10.26803/ijlter.22.6.31

Moritz, A., & Youn, S. yi. (2022). Spatial ability of transitioning 2D to 3D designs in virtual environment: understanding spatial ability in apparel design education. Fashion and Textiles, 9(1). https://doi.org/10.1186/s40691-022-00293-w

Ncho, B. E., Kohli, K., Sadri, V., & Yoganathan, A. P. (2022). Effect of Native Aortic Leaflet Geometry Modification on Transcatheter Aortic Valve Neo-sinus and Aortic Sinus Flow: An In-vitro Study. Cardiovascular Engineering and Technology, 14(2), 182–193. https://doi.org/10.1007/s13239-022-00647-x

Nusaibah, N., Pramudya, I., & Subanti, S. (2021). Geometric Thinking Skills of Seventh Grade Students on the Topic of Triangle and Quadrilateral Based on Van Hiele Geometry Learning Theory. Journal of Physics: Conference Series, 1776(1). https://doi.org/10.1088/1742-6596/1776/1/012020

Parr, T., Da Costa, L., Heins, C., Ramstead, M. J. D., & Friston, K. J. (2021). Memory and Markov Blankets. Entropy, 23(9), 1105. https://doi.org/10.3390/e23091105

Polya, G. (1978). How to solve it: a new aspect of mathematical method second edition. In The Mathematical Gazette (Vol. 30, p. 181).

Pujawan, I. G. N., Suryawan, I. P. P., & Prabawati, D. A. A. (2020). The effect of van hiele learning model on students' spatial abilities. International Journal of Instruction, 13(3), 461–474. https://doi.org/10.29333/iji.2020.13332a

Purnomo, E. A., Sukestiyarno, Y. L., Junaedi, I., & Agoestanto, A. (2022). Analysis of Problem Solving Process on HOTS Test for Integral Calculus. Mathematics Teaching Research Journal, 14(1), 199–214.

Purnomo, E. A., Sukestiyarno, Y. L., Junaedi, I., & Agoestanto, A. (2024a). Stages of Problem-Solving in Answering HOTS-Based Questions in Differential Calculus Courses. Mathematics Teaching Research Journal, 15(6), 116–145.

Purnomo, E. A., Sukestiyarno, Y. L., Junaedi, I., & Agoestanto, A. (2024b). Stages of Problem-Solving in Answering HOTS-Based Questions in Differential Calculus Courses. Mathematics Teaching-Research Journal, 15(6), 116–145.

Rejeki, S., Riyadi, & Siswanto. (2021). Analysis of students' problem-solving ability in solving geometry problem. Journal of Physics: Conference Series, 1918(4). https://doi.org/10.1088/1742-6596/1918/4/042075

Rong, L., & Mononen, R. (2022). Error analysis of students with mathematics learning difficulties in Tibet. Asian Journal for Mathematics Education, 1(1), 52–65. https://doi.org/10.1177/27527263221089357

Ryan, L. P., Jackson, B. M., Hamamoto, H., Eperjesi, T. J., Plappert, T. J., St. John-Sutton, M., Gorman, R. C., & Gorman, J. H. (2008). The Influence of Annuloplasty Ring Geometry on Mitral Leaflet Curvature. The Annals of Thoracic Surgery, 86(3), 749–760. https://doi.org/10.1016/j.athoracsur.2008.03.079

Sahrudin, A., Budiarto, M. T., & Manuharawati. (2022). Epistemic Action of Junior High School Students with Low Spatial Ability in Constructing Cube Nets. International Journal of Educational Methodology, 8(2), 221–230. https://doi.org/10.12973/ijem.8.2.221

Salifu, A. S., Yakubu, A.-R., & Ibrahim, F. I. (2018). Van Hiele Geometric Thinking Levels of Pre-Service Teachers ' of E . P . College of Education , Bimbilla-Ghana. Journal of Education and Practice, 9(23), 108–119. https://iiste.org/Journals/index.php/JEP/article/view/44024

Santos-Trigo, M., Espinosa-Pérez, H., & Reyes-Rodríguez, A. (2008). Connecting dynamic representations of simple mathematical objects with the construction and exploration of conic sections. International Journal of Mathematical Education in Science and Technology, 39(5), 657–669. https://doi.org/10.1080/00207390701871465

Setiawan, A., Wardono, Wijayanti, K., Mulyono, & Bishtawi, H. O. (2024). The Mathematical Literacy Process Based on The Students' Mathematical Resilience. Journal of Ecohumanism, 3(7), 2824–2841. https://doi.org/10.62754/joe.v3i7.4420

Siagian, P. P. G., & Sinaga, B. (2019). Analysis of Difficulty of student ' s Geometry on Van Hiele ' s Thinking Theory. International Journal on Emerging Mathematics Education (IJEME), 3(2), 162–176.

Silmi Juman, Z. A. M., Mathavan, M., Ambegedara, A. S., & Udagedara, I. G. K. (2022). Difficulties in Learning Geometry Component in Mathematics and Active-Based Learning Methods to Overcome the Difficulties. Shanlax International Journal of Education, 10(2), 41–58. https://doi.org/10.34293/education.v10i2.4299

Steele, M. D. (2013). Exploring the mathematical knowledge for teaching geometry and measurement through the design and use of rich assessment tasks. Journal of Mathematics Teacher Education, 16(4), 245–268. https://doi.org/10.1007/s10857-012-9230-3

Sukestiyarno, Y. L. (2020). Metode Penelitian Pendidikan (2nd ed.). Unnes Press.

Trimurtini, Waluya, S. B., Kharisudin, Sukestiyarno, Y. L., & Iqbal. (2021). A Systematic Review on Geometric Thinking: A Review Research Between 2017-2021. European Journal of Educational Research, 10(4), 1907–1918.

Trimurtini, Waluya, S. B., Sukestiyarno, Y. L., & Kharisudin, I. (2022). A Systematic Review on Geometric Thinking: A Review Research Between 2017-2021. European Journal of Educational Research, 11(3), 1245–1257. https://www.researchgate.net/profile/Suntonrapot-Damrongpanit/publication/356662582_Effects_of_Mindset_Democratic_Parenting_Teaching_and_School_Environment_on_Global_Citizenship_of_Ninth-grade_Students/links/61a6dda685c5ea51abc0f7b6/Effects-of-Mindset-Dem

Trimurtini, Waluya, S. B., Sukestiyarno, Y. L., & Kharisudin, I. (2023). Effect of Two-Dimensional Geometry Learning on Geometric Thinking of Undergraduate Students During the COVID-19 Pandemic. Journal of Higher Education Theory and Practice, 23(3), 172–182. https://doi.org/10.33423/jhetp.v23i3.5848

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Uwurukundo, M. S., Maniraho, J. F., & Tusiime, M. (2022). Enhancing Students' Attitudes in Learning 3-Dimension Geometry using GeoGebra. International Journal of Learning, Teaching and Educational Research, 21(6), 286–303. https://doi.org/10.26803/ijlter.21.6.17

Yao, Y. (2021). The geometry of three-way decision. Applied Intelligence, 51(9), 6298–6325. https://doi.org/10.1007/s10489-020-02142-z

Yuliardi, R., & Rosjanuardi, R. (2021). Hypothetical learning trajectory in student's spatial abilities to learn geometric transformation. JRAMathEdu (Journal of Research and Advances in Mathematics Education), 6(3), 174–190. https://doi.org/10.23917/jramathedu.v6i3.13338

Zich, U. (2023). Origami and Descriptive Geometry : Tangible Models to Enhance Spatial Skills. Nexus Network Journal, 25, 277–284.