

Contextualization of Augmented Reality Digital Module Instruction: Grounded Theory Study in Examining Van Hiele's Level of Geometric Thinking

Siti Faizah¹, Cholis Sa'dijah², Lathiful Anwar³, Sudirman⁴, Ahmad Bukhori Muslim⁵

Abstract

Van Hiele's thinking level has an important role in understanding, interpreting, and determining the geometric thinking ability. However, the lack of technological products that can support Van Hiele's improvement in thinking level is a challenge in itself. One of the tools that can help prospective teacher students to reach Van Hiele's level of thinking is Augmented Reality technology. Therefore, this study aims to investigate Van Hiele's level of thinking through the intervention of Augmented Reality Digital Module Instruction (ADMI) using grounded theory design. The participants involved in this study were one lecturer and ten prospective teacher students who experienced difficulties in improving the level of Van Hiele geometric thinking. The data collected through observation, tests, and interviews are analyzed qualitatively through the stages of open coding, axial coding, and selective coding. These research found that students who had difficulties in carrying out the first level of Van Hiele could use ADMI to overcome these difficulties; students can construct knowledge about 3D geometry; the success of students in completing the geometry test involves reasoning. This study emphasize that ADMI contributes to geometry learning because it can make it easier for students to reach the lowest level of Van Hiele.

Keywords: ADMI, Geometric Thinking, Prospective Teacher Students, Van Hiele Level.

Introduction

Geometry is one of the inseparable fields of everyday life, as it involves an understanding of shapes and spaces in 2D and 3D. Therefore, Piere Van Hiele and Dina Van Hiele affirm that the importance of teaching geometry is in accordance with the stage of students' cognitive development (Demir et al., 2023). Through Van Hiele's theory, not only does the teaching of geometry become more structured, but it also helps in evaluating the level of students' understanding of geometric concepts (Ersoy et al., 2019). A deep understanding of these geometric concepts will help build students' problem-solving skills, creative thinking, and visual-spatial abilities, which are essential skills in a variety of everyday life contexts and future career fields.

In the context of elementary school, it is often found that students do not have an adequate understanding, especially in 3D geometry (Hwang et al., 2020; Schoevers et al., 2022). One of the reasons is the teacher's limited understanding of the material (Akkurt Denizli & Erdoğan, 2022). This is a serious concern, given that understanding geometry at a basic level is an important foundation for understanding more complex concepts in the future (Alex & Mammen, 2018). Therefore, greater efforts are needed to train prospective elementary school teachers to have a deep understanding of geometry and the skills to teach it effectively to their students.

Mathematics learning in geometry material is a natural means to develop students' reasoning and skills (Yorulmaz & Çilingir Altınar, 2021). The importance of reasoning or thinking skills needs to be studied because it is needed to understand geometric concepts, as well as to train the geometric thinking skills of

¹ Department of Basic Education, Universitas Negeri Malang, Malang, Indonesia, Email: faizah.siti.pasca@um.ac.id.

² Department of Mathematics, Universitas Negeri Malang, Malang, Indonesia, Email: cholis.sa'dijah.fmipa@um.ac.id (Corresponding Author).

³ Department of Mathematics, Universitas Negeri Malang, Malang, Indonesia, Email: lathiful.anwar.fmipa@um.ac.id.

⁴ Department of Mathematics Education, Universitas Terbuka, Tangerang, Indonesia, Email: sudirman.official@ecampus.ut.ac.id.

⁵ Department of English Education, Universitas Pendidikan Indonesia, Bandung, Indonesia, Email: abukhmuslim@upi.edu

prospective teacher students before they become educators. Today, many students still need help understanding and using the principles of geometry (Anwar et al., 2024). Research shows that students face epistemological difficulties when thinking geometrically, especially when switching from 2D to 3D models, calculating cubes, and measuring 3D shape sizes (Sudirman et al., 2023). This epistemological barrier occurs at all levels of geometric thinking according to Van Hiele's model (Kandaga et al., 2022). Therefore, Van Hiele's geometric thinking level needs to be integrated in learning strategies that involve problem-solving in 3D space and the use of certain technological products (Naufal et al., 2021; Abidin & Abu, 2021).

The results of research in the last five years show that there are still many prospective teacher students who experience limitations in geometric thinking, because they are only able to reach the first three levels of the Van Hiele model and show that prospective teachers are not fully ready to teach geometry (Bonyah & Larbi, 2021). In addition, Mahlaba & Mudaly (2022) concluded that the increase in the level of geometric thinking in the Van Hiele model is highly dependent on the active participation of prospective teacher students when solving problems. Akil et al. in their research found that prospective mathematics teachers still have difficulty improving to the highest level when given inappropriate learning treatment (Akil et al., 2022). Furthermore, Övez & Özdemir (2024) in their research found that most prospective teachers tend to conduct formal evidence but often tentatively use informal proof practices because they have not fully mastered formal evidence. The results of these studies emphasized the need for certain strategies or technologies carried out by teachers to improve the level of geometric thinking (visualization, analysis, abstraction, deduction, and rigor) of prospective elementary school teachers.

One of the ways done by researchers, to improve Van Hiele's thinking level is to use Augmented Reality Digital Module Instruction (ADMI). ADMI is a learning method that combines digital modules with Augmented Reality (AR) technology to enhance the learning experience. The existence of ADMI is expected to provide meaningful information that is easy to receive naturally through the five senses by adding virtual elements. ADMI functions to enrich sensory-motor activities and stimulate the emotional factors of prospective teacher students so that they can be actively involved during the learning process. Motor sensory activities are related to the thinking process of students when constructing knowledge to understand and solve problems. Based on this description, this study aims to investigate the level of geometric thinking of prospective teacher students after using ADMI, which is revealed using grounded theory.

Methods

Design

This study uses grounded theory because the researcher wants to reveal in depth the thinking level of prospective teacher students after using ADMI and the Van Hiele geometric thinking level framework. In addition, the use of grounded theory allows researchers to build theories that are based directly on data collected from participants, thus providing a comprehensive and authentic understanding of the phenomenon being studied (Sudirman et al., 2024). In this context, the use of Van Hiele's geometry level thinking framework is very relevant. This framework consists of five levels of geometric thinking that describe the development of understanding geometry from the most basic to the most complex: Level 1: Visualization - Recognizing geometric shapes based on their appearance. Level 2: Analysis - Understand the properties and components of geometric shapes. Level 3: Abstraction – Able to identify relationships between traits and use informal logic. Level 4: Deduction - Able to understand and construct deductive arguments as well as axiomatic systems. Level 5: Rigor - Have the ability to think rigorously by understanding the formalities and full logic of geometry.

Participant

Participants in this study are one lecturer of the mathematics teaching material development course and ten prospective elementary school teacher students. Participants have an important role in producing prospective teacher students who master geometry material so that there are no more elementary school teachers who have difficulty teaching mathematics subjects. ADMI-based geometry learning is used to make it easier for prospective students to understand 3D objects, as well as to help students to reach the level of Van Hiele geometric thinking. The selection of prospective teacher students as participants is based on the purposive sampling technique because of the ease of access possessed by researchers (Barrat et al., 2015).

Data Collection

The instruments used in the collection of research data are observation, written tests, and interviews. Observation is used to observe the geometry learning process based on the Augmented Reality Digital Instruction Module. The test was used to identify the geometric thinking ability of prospective teacher students who were reviewed from the Van Hiele level. Then the interview was used to deepen the information obtained from the results of observations and geometric thinking tests. Interviews were conducted in a semi-structured manner to lecturers and students who are prospective elementary school teachers. The key questions presented were: (1) how do lecturers and students respond to AR-based geometry learning?; (2) how are the thinking activities of prospective teacher students in solving geometry problems?; (3) do students experience difficulties when solving geometry problems without the help of ADMI?; (4) can all students reach all five levels of Van Hiele's thinking?

Data Analysis

The researcher made observations during the learning process accompanied by small notes related to the activities of lecturers and prospective teacher students. At the end of the learning using ADMI, students were given a 3D geometry test to identify Van Hiele's geometric thinking level. Interviews were conducted with lecturers and prospective teacher students as a triangulation of data suitability. Furthermore, the data was analyzed using three stages, namely open coding, axial coding, and selective coding. The process can be seen in Figure 1.

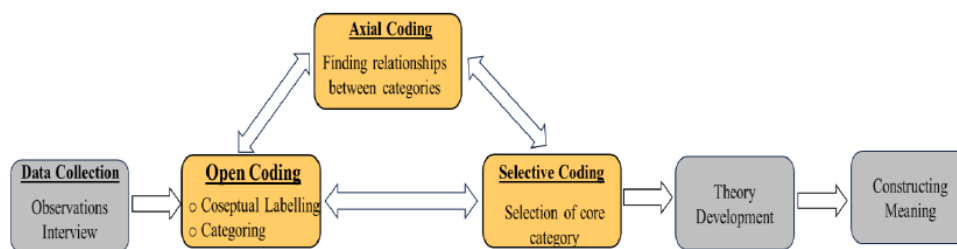


Figure 1. Data Analysis

In the open coding stage, the text of the interview results is analyzed step by step to identify relevant concepts and categories. Then at the axial coding stage, the categories that emerged from the interview results were analyzed to find out the relationship between the two. In the selective coding stage, the categories and dimensions that emerge are considered, compared, and combined to form the final category as a finding.

Results and Discussion

Implementation of Augmented Reality Digital Module Instruction (ADMI)

In the initial stage, the researcher coordinated with the lecturer of the school mathematics teaching materials development course to determine the time for the implementation of learning using digital-based AR or also known as ADMI. Then the right time was obtained to implement ADMI in learning geometry materials. Before the learning process begins, lecturers and researchers ensure students' initial abilities related to geometry materials that have been studied at the previous level. This activity aims to make it easier for students to understand 3D geometry materials using ADMI. During the lesson, the lecturer asked students to open ADMI-based geometry teaching materials and prepare smartphones.

Each student can use this teaching material with the help of a smartphone so that they can carry out the instructions in the book. The teaching materials already contain instructions on how to use them so that lecturers only ask students to study based on the instructions contained in the book. To ensure students' initial abilities, lecturers and researchers give students trigger questions related to the properties and elements of 3D geometry. After that, lecturers and researchers connect students' basic skills with 3D materials to be studied next. ADMI can help students understand the properties and elements of 3D geometry by pointing the smartphone camera as shown in Figure 2.

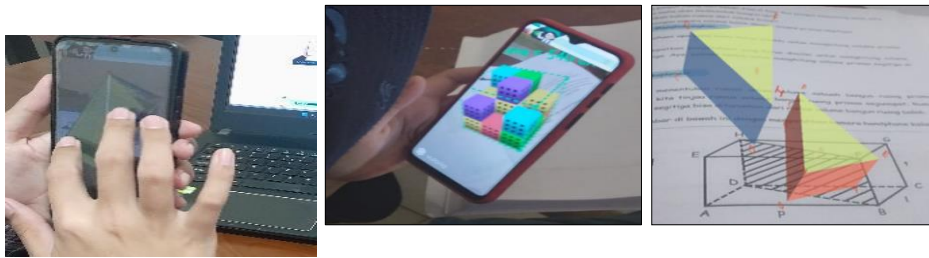


Figure 2. ADMI Exploration

The characteristics of AR can significantly help students understand the shape, structure, and properties of geometric objects by using complex spatial abilities through an in-depth visualization process. This is supported by the theory that digital-based AR can support the development of spatial reasoning skills through the manipulation of 3D objects and the exploration of geometric shapes in real life (Istanti Suwandayani et al., 2023). In this case, students can build their knowledge of 3D geometry based on the results of their exploration using ADMI. After conducting the exploration, students can observe the arrangement of cube units in the form of 3D that appear on the smartphone screen, as shown in Figure 3. Then students are asked to mention the number of cube units, unit size, two-dimensional drawings, or volume. This is done to determine students' initial thinking ability in understanding 3D geometry.

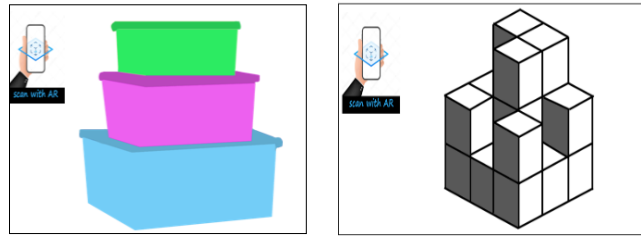


Figure 3. 3D Objects

Based on the results of the exploration, as seen in Figure 2 and Figure 3, it is known that digital-based AR or ADMI can be used to practice deep visualization skills and can improve students' understanding of 3D geometry concepts. In 3D geometry, visualization is the primary ability to understand the shapes, structures, and properties of more complex spatial objects. With the existence of ADMI, it can make it easier for students to see geometric objects in three-dimensional form in real life. It was as if the object was right in front of the eyes. This digital-based AR application can also make it easier for students to understand because students can easily rotate, zoom, or manipulate 3D objects. This provides an opportunity for students to observe every aspect of a two-dimensional object that is difficult to achieve using images. For example, to observe 2D objects present in 3D objects and how their surface structure is, the existence of ADMI can give them a more detailed insight into the structure of an object.

The results of interviews with lecturers also show that ADMI can help students understand 3D geometry material. Because at this time there are still many prospective elementary school teacher students who have difficulty learning geometry because it is difficult to visualize 3D geometry in real life. With the existence of ADMI, it can be a solution to make it easier for students to illustrate or visualize 3D objects. This can be seen from the excerpt of an interview between the researcher (R) and the lecturer (L).

R : *Is this digital-based AR can help you teaching 3D geometry material?*

L : *Of course. I feel that AR application is very helpful when teaching geometry material because students can use their smartphones for the learning process. So far, students have studied mathematics with a discussion and assignment system so that it is not possible to know which students really understand the material and which do not understand the material. With this digital-based AR, it can help me give students an understanding of 3D geometry because it is easy to use to visualize.*

Lecturers also find it easy when using ADMI during learning because it already contains instructions that students can follow easily. Students can easily visualize 3D objects through exploration by sliding the smartphone screen. So far, students have found it difficult to imagine 3D objects because they are abstract without using assistive devices. With ADMI, it can make it easier for prospective teacher students to understand 3D geometry so that later there will be no more elementary school teachers who have difficulty teaching geometry material.

R : *Do you think this digital-based AR is in line with the needs of today's students?*

- L : *Yes... This AR can make learning more interesting because it contains virtual objects that cannot be imagined abstractly by prospective elementary school teacher students. The existence of this AR can also motivate prospective elementary school teacher students to learn mathematics related to geometry materials. Students can follow the instructions in the book and can visualize them with the help of the ADMI application on smartphones.*
- R : *Do you think there are any special characteristics that can make it easier for students to visualize 3D objects?*
- L : *In my opinion, the features in AR that allow students to interact with themselves. Students can interact directly with learning materials through their smartphones related to 3D models, students can rotate or slide the smartphone screen to determine the elements such as sides, angles, diagonals and so on. This will be difficult for students to do if they only imagine abstractly without any tools.*

The results of interviews with lecturers show that ADMI contains geometry material related to real life, and virtual elements in ADMI can make it easier for students to operate. This is in line with the theory that digital-based AR aims to enhance information through virtual elements so that it can provide complete meaning that is impossible to obtain naturally (Rossano et al., 2020). ADMI is a type of technology that can enrich the real world through computer-generated content. Additional content can provide users with an understanding or insight into the real world, 2D and 3D shapes, text, audio, and video. This tool can also increase students' knowledge and perception of something happening in the surrounding environment related to geometry (Elsayed & Al-Najrani, 2021).

ADMI can be a digital-based cognitive assistance system related to the spatial ability to understand geometric objects. The existence of this digital-based cognitive assistance system can make it easier for prospective students to make decisions when solving geometry problems and help them learn new tasks (Eversberg & Lambrecht, 2023). In line with this, Tall formulated the use of concepts in solving mathematical problems in the form of geometric objects consisting of: points, lines, triangles, circles, congruent triangles, and parallel lines that have the properties of Euclid's proof (Tall, 2014; Tall et al., 2013). Therefore, the existence of ADMI can help prospective students in understanding 3D geometric objects so that students can reach the level of Van Hiele geometric thinking when learning or solving geometry problems.

Van Hiele Geometric thinking (VHGT) Test

Exploration of the geometric thinking level of prospective elementary school teacher students was carried out through written tests and in-depth interviews. This test is structured based on Van Hiele's five levels of geometric thinking. Level 1, questions about visualizing 3D objects. Level 2, identifying the properties of geometric objects. Level 3, knowing the relationships between geometric objects based on axiomatic knowledge. Level 4, questions related to logical deduction. Level 5, reasoning. The test results show that not all students are able to do Van Hiele's geometric thinking level because there are students who are not able to visualize 3D objects. There are three categories of geometric thinking levels for prospective elementary school teacher students when completing the test. Three students are included in the high level category because they are able to do all five levels of geometric thinking. There are four students who are included in the medium level category because they are able to do 4 levels, and there are three students who are included in the low level category because they are unable to do Van Hiele's first level, namely visualization.

Students with low level categories still have difficulty solving problems related to visualizing 3D objects without the help of ADMI. Therefore, the subject of this research is students with a low level category because visualization of geometric objects is Van Hiele's lowest level and is very important for prospective teacher students to have. In this case, the students who were selected to be the subject of research with the initials SM. The visualization test questions that SM subjects do are as follows:

"Take a look at the 3D geometry image below!

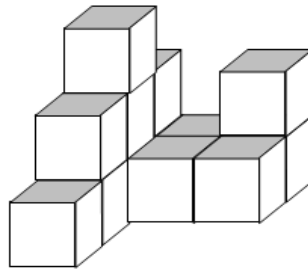


Figure 4. 3D Geometry

Observe the image and then draw the front, side and top view of the arrangement of the cubes!"

Students with a low level category have not been able to visualize the 3D objects in the question. This can be seen from the results of SM's work, which depicts each side in a three-dimensional form as shown in Figure 5.

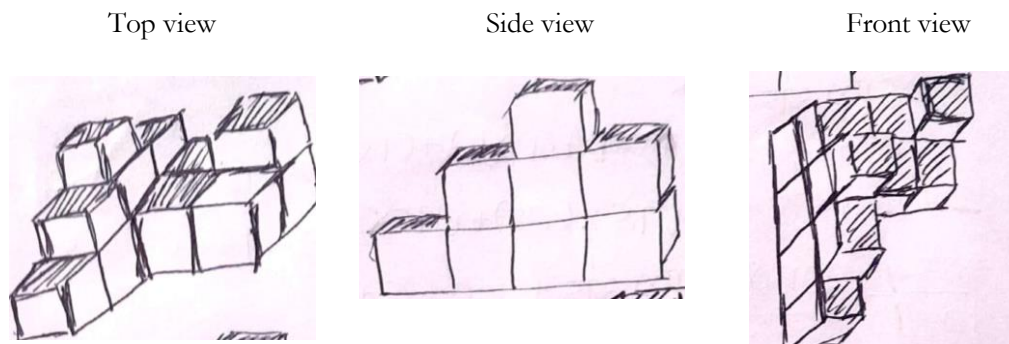


Figure 5. Results of the subject's work

Figure 5 shows that SM subjects have not been able to separate the 2D objects present in the 3D objects. The subject sees the image from above in the form of a 3D image so that he depicts the top view exactly as the question. Similarly, with a side-view image, the subject assumes that the side-view image remains in 3D because he is looking from the side. From the side it contains three piles of cubes, while the bottom one has four cubes, the middle part has three cubes, and the top has one cube so that the illustration of the image remains in the form of a 3D image. As for the front view, it is also depicted in 3D because SM thinks that every side in the picture in the question is a 3D object, so even if viewed from any side, it is still a 3D image as seen from the following interview results.

R : *What causes you to draw the front view, side view, and top view still in 3D images?*

SM : *Because I think the image in the question is 3D geometry so that even if I look at it from any side, it is still a three-dimensional image. So I depict the front view, top view, and side view in the form of a 3D image*

R : *Do you have any difficulties when drawing each side?*

SM : *Ehm....how is it... Actually, when I understood the problem, it was a bit difficult because I had to think abstractly about 3D images, while learning geometry material, it was done using AR applications. If you use an AR application, it's easy to visualize it. However, when working on this problem, you should not use any tools.*

The researcher conducted in-depth interviews with SM subjects regarding their answers to obtain complete data. Interviews are conducted outside lecture hours so that the subject feels comfortable so that the data needed can be completed according to what is needed. The researcher conducted in-depth interviews to strengthen the correctness of the observation results during learning and test results. Based on the results of interviews with the subjects, it can be seen that there is open coding related to geometric thinking activities when learning with lecturers. These codes include the activities of classifying and separating 2D objects contained in 3D images, identifying the properties and elements of 3D geometry, making relationships between objects, and formulating meaningful definitions of geometric objects.

The results of the subject's work showed that he had not been able to solve problems related to the visualization of 3D objects. This difficulty lies in visualizing 3D objects that are top-viewing, side-viewing, and front-viewing. The subject still has difficulty separating 2D objects in 3D. This can be seen from the expression of the subject SM who said that it is difficult to imagine 3D objects without the help of ADMI because using ADMI it is easy to separate 2D objects in 3D. Therefore, the researcher asked the subject to work on the problem by referring to ADMI. The activities that the subjects did when retaking the test were: (1) the subjects opened the AR-based textbook as an aid to recall how to visualize 3D objects; (2) looking for materials related to the visualization of 3D objects; (3) pointing the smartphone screen to a 3D image as during learning; (4) re-observing the picture in the question; (5) identify the 2D objects in the problem; (6) depicts 2D objects that exist in 3D from the front, top, left side, and right side. The subject said that textbooks and AR applications are only an aid to recall how to visualize 2D objects in 3D images. However, when retaking the test, the subject did not use a smartphone because there was no QR code as in the book.

R : *Why is the image different from the previous one?*

SM : *Yes, ma'am. It turns out that the image in the question contains a 2D shape.*

R : *How do you visualize so that you can get a 2D image?*

SM : *After I looked back at the textbook and the AR application, it turned out that the 3D image contains a 2D image, so I need to separate the 2D objects in the problem. I depict the top view, the side view, and the front view.*

I depict the side view from two sides, namely the right and the left, because the image is different, depending on which point of view I visualize it.

R : *How do you visualize 2D images?*

SM : *I opened the textbook, ma'am. Then, I searched for materials related to 3D image visualization. I was looking for material that contained an image of a cellphone or a QR code image so that I could point the cellphone screen at that image. From the cellphone screen, pieces of 2D images in 3D can appear. Well... From that piece of image, I can already imagine the image in the question. After that, I observed the image in the question then I drew the top view, front view, left and right side view in 2D form.*

R : *Oh... like that... So, do you think this digital-based AR can help you visualize 3D geometric objects?*

SM : *I think it is possible because with AR it can make it easier for me to visualize 2D objects in 3D objects.*

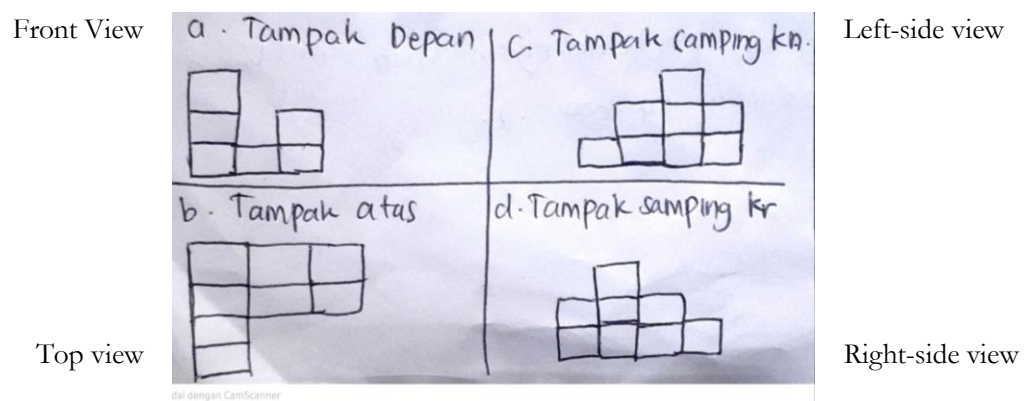


Figure 6. Result of the Subject Work based on ADMI

Figure 6 shows that the subject is able to separate the 2D objects present in the 3D image. The subject admitted that this ability emerged when he used ADMI as a reference to recall how to visualize 3D objects. The subject depicts the front view with square pieces arranged into three, the bottom side as a base consisting of three squares then continued with the next square stack. As for the side view image, two images are given consisting of a right side view image and a left side view image. Then for the top view image, it is presented in the form of a square stack of four. Thus, the subject uses his spatial abilities to visualize the 2D objects contained in the 3D image.

The student's ability to visualize 3D objects can be said to have reached Van Hiele's lowest level. This is in accordance with the theory that the first level of Van Hiele's level is Visualization, which is characterized by the ability of students to assess shapes based on their appearance (Bonyah & Larbi, 2021). This level can

be characterized by: (a) the activity of classifying and separating an object; (b) students need 2D or 3D objects to be able to draw, shape, disassemble, unify, and complete the pieces of the picture. The students' activities are based on the characteristics of different objects so that they can gain an understanding of geometry; (c) to help students from level 1 to level 2, it is necessary to be given challenging questions (Ersoy et al., 2019). At the visualization level, 3D objects can be visually compared by paying attention to certain elements such as vertices, edges, faces, and others (Akkurt Denizli & Erdoğan, 2022).

Second level is Analysis, which is student activity characterized by their ability to recognize or identify geometric objects so that students can classify based on the properties that have been understood (Bonyah & Larbi, 2021). In this case, the student thinks of all forms of building based on the pictures shown to him. For example, students are not only talking about a specific rectangle but can also generalize all 2D or 3D shapes that involve a rectangle. At this level, 3D objects can be compared according to their characteristics, such as angle size, edge length, parallelism, and others. Then the third level is abstraction, students can identify the properties of shapes and realize the relationship between one form and another. For example, there are four right angles, then the building is rectangular but if the shape is square, then all the corners are vertical. At this level, students can understand logical arguments by making informal conclusions regarding the characteristics and properties of geometric shapes. The proofs made by students are intuitive and are not achieved step by step, but students also understand logical arguments to get accurate results (Demir et al., 2023). At this level, 3D objects and their properties can be analyzed based on representations or mathematical structures. Students can connect elements (faces, edges, vertices) on 3D objects (Kandaga et al., 2022).

The fourth level is deduction, i.e. students can think about the properties of building space and its relationship between buildings. Students understand and formulate the definition of geometric building in a meaningful way. If at level 3 students are guided by a system consisting of axioms, definitions, and theorems, then at level 4 students think abstractly related to properties to get logical and unintuitive proof results (Demir et al., 2023). Students are guided by previously proven theorems, they can bring more evidence from other theorems and can understand the process using the induction method (Ersoy et al., 2019). The last level is rigor, this is the highest level in thinking geometry because students study not only one object in geometry but also other objects. Therefore, this level is appropriate for prospective teacher students because students can already think about the similarities and differences between axiomatic systems (Övez & Özdemir, 2024).

Based on the description above, it can be seen that there is a correspondence of data sourced from the results of interviews with lecturers and prospective elementary school teachers regarding the integration of ADMI into the geometric thinking process reviewed from the Van Hiele level. Here are some findings from this study: (1) students have difficulty completing tests related to 3D object visualization without using ADMI; (2) previous learning experience about 2D geometry is a provision for studying 3D objects; (3) the success of students in completing the test involves deductive reasoning. Almost all students said that ADMI can make it easier for them to visualize 3D objects because they have been abstract. Students also said that the experience of learning 2D objects is the capital to study 3D objects. Learning geometry using ADMI can also be a provision of knowledge to learn more complex geometry materials. Therefore, the ADMI-based geometry learning process can make it easier for prospective teacher students to learn 3D objects, and can help students to reach the level of Van Hiele's thinking.

Students in the low-level category no longer have difficulty when doing Van Hiele's lowest level, namely visualizing 3D objects. This is supported by the theory that digital-based AR is a powerful tool to strengthen spatial visualization (Flores-Bascuñana et al., 2020). Spatial visualization is a set of mental skills that allow

individuals to act to represent geometric objects. Manipulation of 3D geometric shapes through ADMI can improve spatial visualization capabilities through virtual elements developed to demonstrate geometric properties (Gutierrez, 1990).

Based on the results of the investigation to the participants, namely lecturers and students, it shows that there is a data consistency in the form of: learning geometry using digital-based AR can make it easier for students to visualize geometric objects. Not only that, students can also reason when completing the written test, this can be seen from the students' answers when completing the test and when answering interview questions. Students give plausible reasoning answers during the interview process. The findings are supported by a theory that says that geometry can help students improve various skills such as deductive reasoning, imaginary visuals, logical arguments, and verification. Basically, in studying geometry, students are not only asked to find the correct answer but also the process to find the result (Firmansyah et al., 2022). Students carry out the reasoning process not only when completing the geometry problem test but also during the interview so that the answers given make sense (Marsitin et al., 2022). Therefore, the results of this study can be formulated by grounded theory as shown in figure 7.

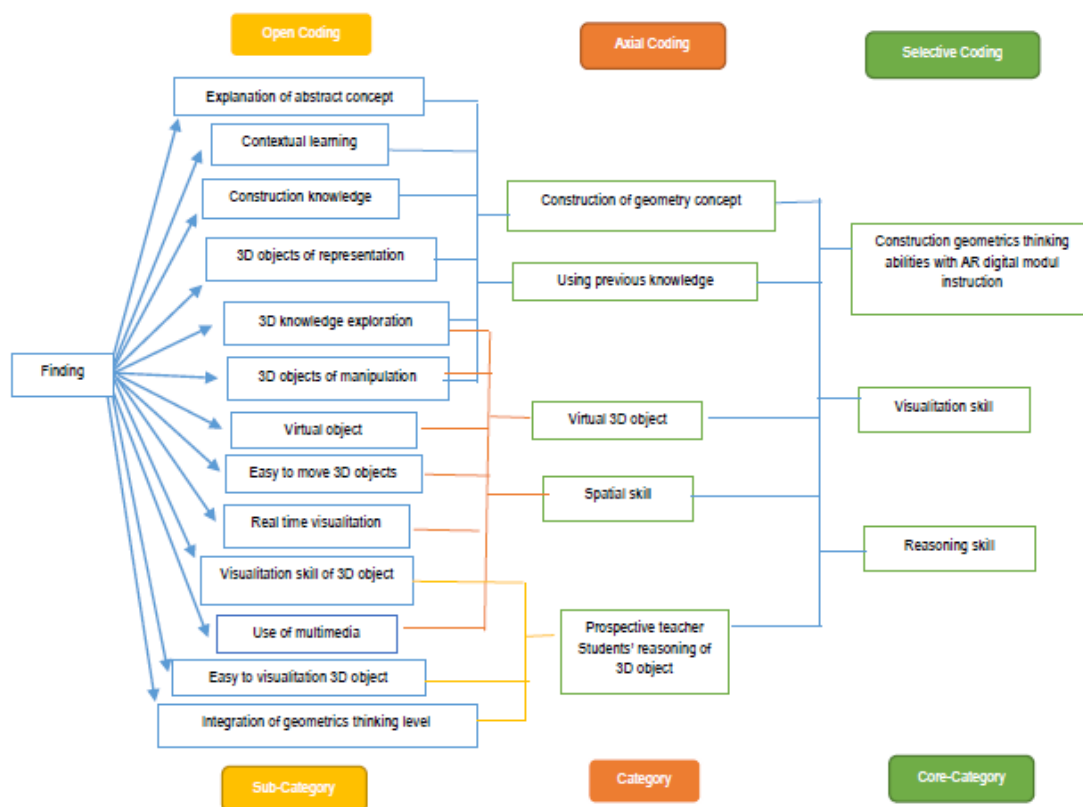


Figure 7. Contextualization of ADMI to Geometric Thinking

From Figure 7, it can be seen that at the stage of open coding, digital-based Augmented Reality can make it easier for lecturers to provide understanding to prospective elementary school teacher students. With ADMI, students can construct knowledge about two-dimensional geometry to study three-dimensional geometry. Students can also easily visualize 2D objects contained in 3D objects with the help of digital-based Augmented Reality. Then, in axial coding, students can build knowledge about 3D geometry based on previous knowledge. Students are also able to reason about geometry using spatial skills. Therefore, at the selective coding stage, it was found that the existence of Augmented Reality Digital Module Instruction (ADMI) can help students reach the lowest level of Van Hiele, namely visualization skills. Students can also do reasoning. Reasoning or thinking process is an important activity in solving mathematical problems (Faizah et al., 2022; Sa'adah et al., 2023). Reasoning has a major position in terms of solving geometry problems because the thinking process contains a mapping between the structures that students already have and other structures that must be inferred based on the equations (Shodikin et al., 2023).

Conclusion

Based on the findings and discussions, it shows that Van Hiele's thinking level can be used to identify students' thinking skills in understanding or solving geometry problems. Currently, students are still having difficulty solving geometry problems related to Van Hiele's level 1, namely 3D object visualization. Students admitted that it was easier to solve problems related to 3D geometry visualization if they referred to digital-based AR. Augmented Reality Digital Module Instruction (ADMI) emerged as a powerful learning design that can help students reach Van Hiele's thinking level. Students can do the first level of Van Hiel, namely 3D object visualization. Students understand 3D geometry by constructing their knowledge, using spatial skills, reasoning, and also based on their experience when learning 3D geometry using ADMI.

The results of this study mean that ADMI has an important role in the Van Hiele geometric thinking level because it can help the difficulties of prospective teacher students in doing Van Hiele level 1, namely geometry visualization. ADMI can facilitate the geometric thinking process of students when studying geometry materials, especially 3D. The experience of students when learning geometry using ADMI is a provision to solve more complex geometry problems related to reasoning. The existence of ADMI also provides experience and provides students to learn more complex geometry materials.

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